Interpolating Sets in the Maximal Ideal Space of H^∞

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1. Introduction

Let H^∞ denote the algebra of all bounded analytic functions on the open unit disc $\mathbf{D}$. Let (z_n) be a sequence of points in $\mathbf{D}$. We shall call (z_n) an *interpolating sequence* if, for each bounded sequence of complex numbers (w_n), there exists a function $f \in H^\infty$ such that $f(z_n) = w_n$ for every $n \in \mathbf{N}$. Let

$$(1) \qquad \delta = \inf_k \prod_{\substack{j=1 \\ j \neq k}}^{\infty} \left| \frac{z_k - z_j}{1 - \overline{z_j} z_k} \right|.$$

We call δ the *separating constant* of (z_n). Carleson's theorem [2] states that a sequence (z_n) in $\mathbf{D}$ is an interpolating sequence if and only if its separating constant δ fulfills $\delta > 0$. The purpose of this paper is to study a natural generalization of this interpolation problem. Let $M(H^\infty)$ denote the maximal ideal space of H^∞, that is, the space of all complex homomorphisms of H^∞, provided with the Gelfand topology. The corona theorem states that $\mathbf{D}$ is dense in $M(H^\infty)$. For $f \in H^\infty$, its Gelfand transform $\hat{f}$ is a continuous function on $M(H^\infty)$ which extends f. When it cannot cause any confusion we will usually omit the distinction between a function and its Gelfand transform. As usual, the pseudohyperbolic distance $\rho(m_1, m_2)$ for $m_1, m_2 \in M(H^\infty)$ is defined by

$$\rho(m_1, m_2) = \sup\{|f(m_1)| : f \in H^\infty,\ \|f\|_\infty \le 1,\ f(m_2) = 0\}.$$

Let $m \in M(H^\infty)$. The set

$$P(m) = \{m' \in M(H^\infty) : \rho(m, m') < 1\}$$

is called the *Gleason part* of m. If $P(m)$ contains at least two points, m is called a *nontrivial* point.

NOTATION. We will denote the set of all nontrivial points by G.

For $E \subset M(H^\infty)$, let $C(E)$ denote the set of all continuous functions on E. To generalize the interpolating problem we introduce the following concept.

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