The Null Blow-Up of a
Surface in Minkowski 3-Space and
Intersection in the Spacelike Grassman

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Introduction

A smoothly immersed compact orientable surface Q in 3-dimensional Minkowski space can be decomposed into a disjoint union $D^+ \cup D^0 \cup D^-$, where the induced metric is positive definite on D^+, degenerate on D^0, and indefinite on D^-. Along D^0, the line orthogonal to Q is also tangent to Q. Imposing natural transversality conditions on this configuration stratifies the surface so $SP \subset D^0 \subset Q$, where the set SP of stall points is contained in the set D^0 of stall curves that is embedded in Q. The stratification is defined as the loci, where the orthogonal line bundle is tangent to the next lower stratum. In [K2] we constructed a Gauss map for Q into the 2-sphere, $g: Q \to S^2$, with degree $\pm \frac{1}{2}\chi(Q)$. Here we construct a Gauss map for D^0 into the compactified spacelike Grassman, $cg: D^0 \to S^1 \times S^1$. In this context points of SP correspond to intersection points of cg with the diagonal in $S^1 \times S^1$. In this paper we establish a formula relating: the degree of g, $\chi(D^+)$, and the interesection number of cg with the diagonal (Theorem 4). As a consequence we have two integral inequalities that can be used to characterize simple configurations (Theorem 6). We then construct the null blow-up NB of Q. This is a compact folded double cover of $D^- \cup D^0$, $\rho: NB \to D^- \cup D^0$, with an oriented line field L whose zero points are exactly $SP \subset D^0$. This null blow-up is completely determined by the first fundamental form on Q, and can be thought of as a completion space for null geodesics in Q (i.e., a blow-up space for the singularities in the null geodesic ODE). We then show that the sum of the indices at these zero points is the intersection number of cg with the diagonal in $S^1 \times S^1$ (Corollary 8). Since this line field ρ-projects to null subspaces in D^-, this corollary links purely extrinsic properties (a Gauss map for D^0) with purely intrinsic properties (the global dynamics of null pre-geodesics in $D^- \cup D^0$). Furthermore, since these zero points can be viewed as a degenerate type of "conjugate point", we have a new link between conjugacy in the null geodesic ODE and global properties of the underlying manifold.