Boundary Behavior of Certain Holomorphic Maps

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1. Introduction

Our point of departure is the recent work of Alinhac, Baouendi, and Rothschild [3] and of Bell and Lempert [4] on the boundary behavior of holomorphic maps from $\mathbf{C}$ to $\mathbf{C}^n$. In the scalar case $n=1$, the results may be formulated as follows. Let H_r denote the intersection of the open disk of radius r centered at the origin with the upper half-plane, and let σ_r denote the closed semi-circle in its boundary.

THEOREM. *Let Γ be a smooth Jordan arc in $\mathbf{C}$. Let f be a holomorphic function on H_r such that the cluster set of f along $[-r,r]$ is contained in Γ. Then*

(a1) *f extends to be continuous on $(-r,r)$,*
(a2) *f is smooth on $(-r,r)\cup H_r$, and*
(b) *f has finite order at each point of $(-r,r)$ unless f is constant.*

Part (a1) is not explicitly stated in [4] but follows from the argument there because the classical reflection principle yields such continuity. The meaning of "smoothness" is $\mathcal{C}^\infty$ for f and Γ in [4], while [3] treats f in Lipschitz spaces and Γ being $\mathcal{C}^k$ with $k\geq 2$. In the former case, "finite order" at x simply means that some derivative $f^{(N)}(x)\neq 0$; in the latter case it means not of infinite order, that is, $f(z)-f(x)=O((z-x)^N)$ does not hold for every N.

The main objective in the cited work is to handle higher-dimensional mappings where Γ is replaced by a totally real manifold. The first results of this type were due to Chirka [6]; previous work has also been done by Rosay [12] and Pinchuk and Khasanov [10]. However, according to [3], the unique continuation property (b) is new even in the scalar case. It is proved in [3] and [4] by PDE methods. We first consider the case when Γ is not assumed to be smooth but is just a (continuous) Jordan arc. It turns out that a sort of finiteness (b1) still holds, with no assumption of smoothness.

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