Crossed Product Criteria and Skew Linear Groups II

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This paper concerns the class $\langle P, L\rangle(\mathfrak{A}\mathfrak{F})$ of groups built from the classes $\mathfrak{A}$ of abelian groups and $\mathfrak{F}$ of finite groups by repeated use of the local and poly operators L and P, transfinitely if necessary. The nature of the paper makes the use of Hall's calculus of group classes (see the opening pages of [3]) more of a necessity than a convenience. Our previous paper [7] with this title concerned the smaller class $\langle P, L\rangle\mathfrak{A}$.

Throughout this paper F denotes a (commutative) field, D a division F-algebra and n a positive integer. For any group G, $\tau(G)$ is the unique maximal locally finite normal subgroup of G, $\eta(G)$ the Hirsch–Plotkin radical of G, $\zeta_1(G)$ the centre and $\zeta_2(G)$ the second centre of G, and $\alpha(G)$ and $\beta(G)$ are defined by

$$\beta(G)/\tau(G) = \eta(G/\tau(G)) \quad \text{and} \quad \alpha(G)/\tau(G) = \zeta_1(\beta(G)/\tau(G)).$$

The core of our main result here, of which there are many corollaries, can be summarized as follows.

Let G be a primitive subgroup of $GL(n, D)$ with $G \in \langle P, L\rangle(\mathfrak{A}\mathfrak{F})$. Then the F-subalgebra $F[G]$ of the full matrix ring $D^{n\times n}$ generated by G is a crossed product over the locally-finite by abelian normal subgroup $\alpha(G)$ of G; that is, $F[G]$ is free as a left and right $F[\alpha(G)]$-module on any transversal of $\alpha(G)$ to G.

The proofs below depend heavily on the results and proofs of [7]. One difference perhaps we should specifically mention at the outset. Unlike [7], the major results here depend ultimately on the classification of the finite simple groups (in the weak form, that there exist only a finite number of sporadic groups). In this sense [7] and the present paper operate at different levels. When [7] was written I did not believe that comparable results for $\langle P, L\rangle(\mathfrak{A}\mathfrak{F})$ existed. In one way this is true: Unlike $\langle P, L\rangle\mathfrak{A}$-groups, $\langle P, L\rangle(\mathfrak{A}\mathfrak{F})$-groups in general do not have a "Zalesskiĭ" subgroup; specifically, our F-algebras need not be crossed products over normal FC-groups, canonical or otherwise. However the partial results obtained by working directly with $\alpha(G)$ do hold and are almost as useful.

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