Corona Theorems for Subalgebras of H^∞

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Let H^∞ be the Banach algebra of all bounded analytic functions in the open unit disk $\mathbf{D}$. The famous corona theorem of Carleson [1] states that the unit disk is dense in the maximal ideal space $M(H^\infty)$ of H^∞. Equivalently, the ideal $I=(f_1,\dots,f_N)$ generated by the functions $f_i \in H^\infty$ $(i=1,\dots,N)$ equals the whole algebra H^∞ if and only if $\sum_{k=1}^N |f_k| \geq \delta > 0$ in $\mathbf{D}$.

Let L^∞ denote the space of essentially bounded, Lebesgue measurable functions on the unit circle T. It is standard to identify, via radial limits, H^∞ with a uniformly closed subalgebra of L^∞. Let B be a Douglas algebra, that is, a uniformly closed subalgebra of L^∞ containing H^∞. Associated with each Douglas algebra is the largest C^*-algebra QB contained in B, that is,

$$QB = B \cap \bar{B} = \{f \in B : \bar{f} \in B\},$$

and the C^*-algebra CB generated by the invertible inner functions in B and their conjugates.

By using the corona theorem for H^∞, Chang and Marshall [2] showed that the unit disk is dense in the maximal ideal space of $CA_B := CB \cap H^\infty$. Later Sundberg and Wolff [15] could prove by highly sophisticated methods that the corona theorem is also true in the algebra $QA_B := QB \cap H^\infty$. It is now quite surprising that the methods of Chang and Marshall [2] not only yield another proof of the corona theorem for QA_B, but that they can be used to show that *every* subalgebra A of H^∞ of the form $A = \mathcal{C} \cap H^\infty$ has the corona property, where $\mathcal{C}$ is a C^*-algebra satisfying $CB \subseteq \mathcal{C} \subseteq QB$. The proof of this result will be a major object of this paper. Incidentally, we obtain some other properties of algebras of this type. This will answer a question of Dawson [3, §6] concerning the ideal structure of subalgebras of H^∞.

The Corona Theorem for Admissible Algebras

DEFINITION 1. Let A be a closed subalgebra of H^∞. According to Metzger [12] we shall say that A has the "weak F-property" if f belongs to A whenever $uf \in A$ for some inner function $u \in A$. If we merely assume that u is an inner function in H^∞, then we say that A has the "F-property" (in the sense of Khavin).

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