COMPLEMENTATION OF PRINCIPAL IDEALS IN WEIGHTED (FN)-ALGEBRAS OF ENTIRE FUNCTIONS

Jürgen Wolter

Introduction. In this paper we study (FN)-algebras

$$A_p^0(\mathbf{C}^N) := A_p^0 := \{f \in H(\mathbf{C}^N) \mid \text{for all } D > 0\colon \sup_{z \in \mathbf{C}^N} |f(z)| e^{-Dp(z)} < \infty\}$$

for nonnegative plurisubharmonic functions p such that $\log(1+|z|^2) = o(p(z))$. For the sake of simplicity we will assume in the introduction that $p(z) = |z|^\alpha$, $\alpha > 0$. Then each principal ideal $I(G)$ generated by $G \in A_p^0$ is closed in A_p^0. Our main result is the following.

THEOREM. *For $G \in A_p^0(\mathbf{C}^2)$ and $G(z,w) := f(z) - \sum_{i+j \le m} a_{i,j} z^i w^j$ ($m \in \mathbf{N}$, $a_{i,j} \in \mathbf{C}$, $a_{0,m} \neq 0$), the following statements are equivalent:*

(1) *f is a polynomial;*
(2) *$I(G)$ is complemented in $A_p^0(\mathbf{C}^2)$ (i.e., there exists a continuous linear projection on $A_p^0(\mathbf{C}^2)$ with range $I(G)$);*
(3) *for some $H \in A_p^0(\mathbf{C}^2) \setminus \{0\}$ the ideal $I(GH)$ is complemented in $A_p^0(\mathbf{C}^2)$.*

For $\alpha > 1$, the A_p^0 are isomorphic to the strong duals of weighted (DFN)-spaces of entire functions by Fourier–Borel transformation, the pointwise multiplication carried over to form the convolution product. Therefore, the results of this paper imply that certain convolution operators have no continuous linear right inverses.

The main point of our theorem is (2) $\Rightarrow$ (1). To prove it, we assume that f is not a polynomial. Note then that $A_p^0(\mathbf{C}^2)$ is a power series space of infinite type. Hence by a theorem of Zahariuta [27], $A_p^0(\mathbf{C}^2)$ cannot contain a subspace isomorphic to a power series space of finite type. We shall find such a subspace E in $A_p^0(\mathbf{C}^2)/I(G)$. However, (2) would imply that $A_p^0(\mathbf{C}^2)/I(G)$, and hence E, are subspaces of $A_p^0(\mathbf{C}^2)$.

We must find E only in the case $G(z,w) = f(z) - w$, as the others follow by a substitution argument. For such G we have a canonical isomorphism of locally convex algebras $A_p^0(\mathbf{C}^2)/I(G) \to A_q^0(\mathbf{C})$, with $q(z) := p(z, f(z))$ by a variant of an interpolation theorem of Berenstein and Taylor [3].

An extension of results of Meise and Taylor [14] shows that, for certain closed ideals J in $A_q^0(\mathbf{C})$, the quotient $A_q^0(\mathbf{C})/J$ is a power series space of finite type. The essential step in the proof is to find such an ideal J that is complemented in $A_q^0(\mathbf{C})$. To do this, we construct first a sequence of subharmonic functions and use them, together with Hörmander's L^2-theory of the $\bar{\partial}$-operator, to find for a certain ideal J a Schauder basis of a complement $E \cong A_q^0(\mathbf{C})/J$.

Received December 4, 1987. Revision received August 1, 1988.
Michigan Math. J. 35 (1988).