NORMAL EXTENSIONS OF SUBNORMAL COMPOSITION OPERATORS

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0. Introduction. A composition operator is an operator C of the form $Cf = f \circ T$, where T is a transformation on the state space X of a σ-finite measure space (X, Σ, m). Various questions of normality and semi-normality of such operators have been addressed. Usually the Radon–Nikodym derivatives $dm \circ T^{-n}/dm$ play a central role in these investigations. In this article it is shown how a subnormal composition operator may be extended to a normal composition operator. Section 1 deals with general properties of composition operators and re-states some known properties of composition operators regarding normality and semi-normality. Section 2 is concerned with establishing the extension of a subnormal composition operator to a quasi-normal composition operator. It is shown that if T is invertible (and bi-measurable) then this construction yields the minimal normal extension of C. The material in Section 3 relies heavily on a modification of an ergodic theory technique for constructing an invertible transformation in terms of T. This material is then used to construct a minimal normal composition operator extension of an arbitrary subnormal composition operator.

1. Preliminaries. Let (X, Σ, m) be a σ-finite measure space and let T be a mapping of X onto X such that $T^{-1}\Sigma \subseteq \Sigma$. The linear transformation C on $L^2_m = L^2(X, \Sigma, m)$ given by $Cf = f \circ T$ is called the *composition operator* induced by T. General properties of composition operators may be found in [7]. In particular, C is a bounded operator on L^2_m if and only if $m \circ T^{-1}$ is absolutely continuous with respect to m and the Radon–Nikodym derivative $dm \circ T^{-1}/dm$ is essentially bounded. Let $h = dm \circ T^{-1}/dm$. These assumptions will be made throughout the remainder of this article. Conditions for composition operators to belong to certain specific classes of operators have been widely studied. Proposition 1.1 below lists those results pertinent to this article together with references. The following notation will be used.

(i) For $f \in L^2_m$ or $f \geq 0$ a.e. dm, $E(f) = E(f \mid T^{-1}\Sigma)$ is the conditional expectation of f with respect to $T^{-1}\Sigma$. For $f \in L^2_m$, $E_n(f)$ is then the orthogonal projection $E_n(f) = E(f \mid T^{-n}\Sigma)$.

(ii) $h_n = dm \circ T^{-n}/dm$, $h = h_1$.

1.1. PROPOSITION. (a) *C is normal if and only if $T^{-1}\Sigma = \Sigma$, T is invertible and bi-measurable, and $h = h \circ T$ a.e. dm* ([9], [12]).

(b) *C is quasi-normal if and only if $h = h \circ T$ a.e. dm* ([10], [12]).

(c) *The following are equivalent.*

(i) *C is subnormal.*

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