FRÉCHET ENVELOPES OF CERTAIN ALGEBRAS OF ANALYTIC FUNCTIONS

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1. Introduction. Let D denote the open unit disc in the complex plane. The Smirnov class (or Hardy algebra) N^+ consists of those analytic functions on D for which

$$\lim_{r\to 1-}\frac{1}{2\pi}\int_{-\pi}^{\pi}\log^+|f(re^{i\theta})|\,d\theta=\frac{1}{2\pi}\int_{-\pi}^{\pi}\log^+|f(e^{i\theta})|\,d\theta<+\infty,$$

where $f(e^{i\theta})$ are the boundary values of f on ∂D [2]. Although N^+ has appeared in the classical literature since 1932 (see [2, p. 31]), it was not until the early 1970s that a study of the linear topological properties was carried out by Yanagihara ([12], [13]). He showed in [13] that N^+ is an F-space (complete, metrizable linear topological space), in fact an F-algebra (multiplication is jointly continuous) with the translation-invariant metric d defined by

$$d(f,g)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\log(1+|f(e^{i\theta})-g(e^{i\theta})|)\,d\theta.$$

Like the Hardy spaces H^p, for $0<p<1$, Yanagihara showed that N^+ is not locally convex but still has a rich dual space. However, in contrast to H^p, he showed that N^+ is not locally bounded (i.e., has no bounded neighborhood of zero).

The Fréchet envelope for N^+ was identified by Yanagihara [12] as F^+, those analytic functions on D for which

$$\lim_{r\to 1-}(1-r)\log^+(\max_{|z|=r}|f(z)|)=0.$$

He showed that the topology of F^+ can be given by a family of seminorms, $(\|\cdot\|_c)_{c>0}$, defined by

$$\|f\|_c=\sum_{0}^{\infty}|a_n|\exp[-cn^{1/2}],\quad c>0,$$

where (a_n) are the Taylor coefficients of f. Natural generalizations of N^+ have been studied by Stoll in [11]: $(\mathrm{Log}^+H)^\alpha$, $\alpha>1$, the Hardy-Orlicz algebra of analytic functions on D which satisfy

$$\sup_{0<r<1}\frac{1}{2\pi}\int_{-\pi}^{\pi}[\log^+(|f(re^{i\theta})|]^\alpha\,d\theta<+\infty,$$

and $(\mathrm{Log}^+H(D))^\alpha$, $\alpha\geq 1$, the Bergman-Orlicz algebra of analytic functions on D for which

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