LOCAL INVARIANTS OF FOLIATIONS BY REAL HYPERSURFACES

James J. Faran, V

0. Introduction. In this paper we present complete local invariants under biholomorphic mappings of foliations of complex space by nondegenerate real hypersurfaces. Perhaps the most notable of these invariants is an intrinsic normal direction. Flow along this normal direction provides a map from leaf to leaf which is a contact transformation but, in general, not a CR isomorphism of the leaves of the foliation.

This study of local invariants is intentionally imitative of the work of Chern and Moser [1] — the local invariants of a foliation by real hypersurfaces should be similar to the local invariants of a single real hypersurface. One difference is the existence of an intrinsic normal direction to a foliation. Another involves the fourth-order curvature tensor $S_{\alpha\gamma}{}^{\beta}{}_{\bar\sigma}$. For a single real hypersurface, this has trace zero; that is, the Ricci tensor $S_{\gamma\bar\sigma}=S_{\alpha\gamma}{}^{\alpha}{}_{\bar\sigma}=0$. This is not true for foliations. For example, while the foliation by hyperquadrics $\{\operatorname{Im} z_{n+1}-\sum|z_j|^2=\text{constant}\}$ has curvature zero, the foliation by spheres $\{\sum|z_j|^2=\text{constant}\}$ has positive Ricci curvature. It is natural to ask whether every real hypersurface exists as the leaf of a foliation with vanishing Ricci curvature $S_{\gamma\bar\sigma}$. We provide examples showing that this is not true.

It should be noted that Graham and Lee have studied similar objects. In [2] they examine the geometry of a foliation by real hypersurfaces together with a defining function. The local invariants they obtain are quite similar to those obtained here.

1. Local invariants. Let $\mathfrak{F}$ be a foliation by real hypersurfaces of a neighborhood of a point in $\mathbf{C}^{n+1}$, $n\geq 1$. We assume that $\mathfrak{F}$ is given as the level sets of a $\mathbf{C}^\infty$ real-valued function $r(z,\bar z)$ with $dr\neq 0$. The function r may be replaced by $r^*(z,\bar z)=f(r(z,\bar z))$, where f is any function of one variable with nonzero derivative. In this section, we consider the local equivalence problem for such foliations, to wit: if $\mathfrak{F}$ is a foliation in a neighborhood of a point p and $\mathcal{G}$ is a foliation in a neighborhood of a point q, does there exist a biholomorphic mapping of a neighborhood of p taking p to q and taking the leaves of $\mathfrak{F}$ to the leaves of $\mathcal{G}$? Our solution is obtained via Cartan's Method of Equivalence. This method may be described as follows.

First, we restate the problem as one of equivalence of G-structures. (A G-structure may be viewed as a coframe — a basis for the cotangent space — well-defined up to the action of the group G.) Second, using conditions on the exterior derivatives of the elements of the coframe, we reduce the group to a smaller group. Third, we lift to the principal bundle of all such coframes obtaining a

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