ON THE AUTOMORPHISM GROUP OF HYPERELLIPTIC KLEIN SURFACES

E. Bujalance, J. A. Bujalance, and E. Martínez

1. In [6] *hyperelliptic Klein surfaces* (HKS) were characterized by means of *non-Euclidean crystallographic groups* (NEC groups). An HKS is a Klein surface with non-empty boundary X that admits an automorphism ϕ of order two such that the quotient $X/\langle\phi\rangle$ has *algebraic genus* zero. Then, the automorphism group of X has order $2N$.

Given an HKS X with algebraic genus $p \geq 2$ and k boundary components, and given $|\mathrm{Aut}\, X| = 2N$ (N odd), we obtain in this paper the possible values for N. Moreover, for each one of these values we prove that there exists an HKS X, with the above conditions, such that $|\mathrm{Aut}\, X| = 2N$.

In the particular cases $p=2$ or $p=3$, the list of the automorphism groups is given in [5] and [7].

2. A Klein surface (see [1]) may be expressed as D/Γ where D is the hyperbolic plane and Γ is a certain NEC group [12]. NEC groups were introduced by Wilkie [15]. Macbeath [10] associated to each NEC group Γ a *signature* that has the form

$$(g; \pm; [m_1, \dots, m_r], \{(n_{11}, \dots, n_{1s_1}) \cdots (n_{k1}, \dots, n_{ks_k})\}),$$

and determined the algebraic structure of Γ. In this signature the numbers are integers and $g \geq 0$, $m_i \geq 2$, $n_{ij} \geq 2$. The number g is the topological genus of the group (and that of D/Γ). The sign determines the orientability of D/Γ. The numbers m_i are the *proper periods* and the brackets $(n_{i1}, \dots, n_{is_i})$ are the *period-cycles*. The number k of period-cycles is equal to the number of boundary components of D/Γ. Numbers n_{ij} are the *periods of the period-cycle* $(n_{i1}, \dots, n_{is_i})$.

The *canonical* presentation of Γ is as follows.

Generators

(i) x_i $\quad i=1,\dots,r$
(ii) e_i $\quad i=1,\dots,k$
(iii) c_{ij} $\quad i=1,\dots,k;\ j=0,\dots,s_i$
(iv) a_i, b_i $\quad i=1,\dots,g$ (if sign '+')
$\quad d_i$ $\quad i=1,\dots,g$ (if sign '−')

Relations

(i) $x_i^{m_i}=1$ $\qquad i=1,\dots,r$

(ii) $c_{i,j-1}^2 = c_{i,j}^2 = (c_{i,j-1}c_{i,j})^{n_{ij}} = 1$ $\qquad i=1,\dots,k;\ j=1,\dots,s_i$

Received July 7, 1987. Revision received July 26, 1988.
The authors are partially supported by CAICYT.
Michigan Math. J. 35 (1988).