MAXIMAL SPACELIKE SUBMANIFOLDS OF A PSEUDORIEMANNIAN SPACE OF CONSTANT CURVATURE

Toru Ishihara

1. Introduction. Generalizing Lawson's result [4], Chern–DoCarmo–Kobayashi proved the following in [3]. Let M be an n-dimensional minimal submanifold of a unit sphere S^{n+p}. Let S be the square of the length of the second fundamental form of M. If M is compact, it follows from Simon's result that if $S \le n/(2-1/p)$ everywhere on M then either $S=0$ or $S=n/(2-1/p)$. The Veronese surface in S^4 and $M_{m,n-m}$ in S^{n+1} are the only compact minimal submanifolds of dimension n in S^{n+p} satisfying $S=n/(2-1/p)$, where $M_{m,n-m}$ is the manifold $S^m(\sqrt{m/n}\,)\times S^{n-m}(\sqrt{(n-m)/n}\,)$ which is naturally imbedded in S^{n+1}.

On the other hand, in this paper we investigate maximal spacelike submanifolds of a pseudo-Riemannian space of constant curvature. Let $N_p^{n+p}(c)$ be an $(n+p)$-dimensional pseudo-Riemannian manifold of constant curvature c whose index is p. Let M be an n-dimensional complete Riemannian manifold isometrically immersed in $N_p^{n+p}(c)$. Note that the codimension is equal to the index.

The pseudohyperbolic space of radius r (>0) is the hyperquadric

$$H_p^{n+p}(r)=\{x\in \mathbf{R}_{p+1}^{n+p+1};\ \langle x,x\rangle = x_1^2+\cdots+x_n^2-x_{n+1}^2-\cdots-x_{n+p+1}^2=-r^2\}.$$

This is a space of constant curvature $-1/r^2$. Let $H^n(r)$ be the component of $H_0^n(r)$ through $(0,\dots,0,r)$. Here, we describe two examples of maximal spacelike immersions. We consider the mapping defined by

$$u_1=\frac{1}{\sqrt3}yz,\quad u_2=\frac{1}{\sqrt3}zx,\quad u_3=\frac{1}{\sqrt3}xy,\quad u_4=\frac{1}{2\sqrt3}(x^2-y^2),$$
$$u_5=\tfrac16(x^2+y^2+2z^2),$$

where (x,y,z) is the natural coordinate system in $\mathbf{R}_1^3$ and (u_1,u_2,u_3,u_4,u_5) is the natural coordinate system in $\mathbf{R}_3^5$. This defines an isometric maximal immersion of $H^2(\sqrt3)$ into $H_2^4(1)$. We may call this the *hyperbolic Veronese surface.* Let $n_1,\dots,n_{p+1}$ be positive integers and $n=n_1+\cdots+n_{p+1}$. Let x_i be a point of $H^{n_i}(\sqrt{n_i/n}\,)$. Then $x=(x_1,\dots,x_{p+1})$ is a vector in $\mathbf{R}_{p+1}^{n+p+1}$ with $\langle x,x\rangle=-1$. This defines also an isometric immersion of

$$H_{n_1,\dots,n_{p+1}}=H^{n_1}(\sqrt{n_1/n}\,)\times\cdots\times H^{n_{p+1}}(\sqrt{n_{p+1}/n}\,)$$

into $H_p^{n+p}(1)$. Now, it has been proved by Cheng and Yau [2] that a complete maximal spacelike hypersurface in the Minkowski $(n+1)$-space is totally geodesic (see also [1]). First, we generalize this result slightly.

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