HYPERBOLIC ENDS AND CONTINUA

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1. Introduction. Let F be a closed orientable surface of genus greater than one. The Nielson–Thurston theorem states that every homeomorphism of F is isotopic to a homeomorphism of F that (1) has finite order, or (2) is reducible, or (3) is pseudo-Anosov. The last case is the most common and the most interesting.

The behavior of the isotopy class of a pseudo-Anosov homeomorphism is captured in a unique pair of projective classes of measured laminations preserved by the homeomorphism. The underlying geodesic laminations are indecomposable continua with only points and arcs as subcontinua.

Such a geodesic lamination G is best understood by considering its preimage G in the universal covering space of F, hyperbolic 2-space H. In the Poincaré disc model of H, each leaf of G lifts to a (complete) geodesic in H, an arc of a circle in the Euclidean plane.

How can one see that G is not homogeneous? An interesting answer is to show that if G has the micro-transitivity property of Effros, then so does $\tilde{G}$. This is a contradiction, since close to each point x of $\tilde{G}$ is a point y of $\tilde{G}$ such that the geodesic G_x of $\tilde{G}$ containing x and the geodesic G_y of $\tilde{G}$ containing y are ultraparallel, so no bounded homeomorphism can move G_x onto G_y.

This suggests that if X is a homogeneous curve in F and x is a point of its preimage $\tilde{X}$ in H, then it is possible to assign to x a set of points in the circle at ∞ in such a way that this set is a local invariant of $\tilde{X}$ as well as an invariant of the component of x in $\tilde{X}$. How could one do this for an arbitrary homogeneous curve?

If X is a curve with nontrivial shape and Q is the Hilbert cube, then X has an essential embedding into $F \times Q$. Let $p \times 1 : H \times Q \to F \times Q$ be the universal cover of $F \times Q$, and let $\tilde{X}$ be the preimage of X. If K is a component of $\tilde{X}$, it will be shown that one can associate with K a certain subset $E(K)$ of the circle at ∞; this will be called the set of ends of K.

THEOREM. *If X is a homogeneous curve, then the set $E(K)$ of ends of K is a local invariant of $\tilde{X}$.*

Given any natural number n, there exists a curve X in $F \times Q$ and a component K of $\tilde{X}$ such that $E(K)$ is an n-point set. The same holds for various infinite subsets of the circle at ∞. For homogeneous curves, however, the topological type of $E(K)$ is quite restricted.

THEOREM. *If X is a homogeneous curve, then $E(K)$ is either a two-point set or a Cantor set.*

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