AN OBSTRUCTION TO THE RESOLUTION OF HOMOLOGY MANIFOLDS

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The author's paper [2] asserts that an ENR homology manifold of dimension ≥ 4 has a resolution. Unfortunately there is an error in the proof, and it is currently unknown whether unresolvable homology manifolds exist. The proof does show that there is at most a single integer obstruction, heuristically the index of a 0-dimensional subobject (a "point"). The properties of this obstruction are the subject of this paper.

The obstruction can be roughly described by: let $U \subset X^n$ be an open set with a proper degree 1 map $f: U \to \mathbf{R}^n$. Make f transverse (in a sense to be made precise below) to 0; then the *local index* is defined by $i(X) = \operatorname{index} f^{-1}(0)$. If X is a manifold and we use manifold transversality then $f^{-1}(0)$ can be arranged to be a single point, so $i(X) = 1$. If X is not a manifold then some other form of transversality must be used, and the ones currently available might yield inverses with indices different from 1. This index must be congruent to 1 mod 8, so for example there might be homology manifolds with "points" of index 9.

We discuss the transversality to be used. There is currently no direct transversality construction for homology manifolds, so we pass to Poincaré complexes or chain complexes. In the Poincaré context there is a dimension restriction (≥ 4) for transversality. Therefore we have to raise the dimension, for example by multiplication by CP^2, to apply it to the situation at hand. This leads to an inverse image of positive dimension, which may have a nontrivial index. If chain complexes are used there are no dimension restrictions to transversality, so we can get a 0-dimensional chain complex as the "inverse image" of 0 in $\mathbf{R}^n$. However, 0-dimensional chain complexes can have nontrivial index. In either case the index gives the obstruction.

There are some curious niches in manifold theory for unresolvable homology manifolds. It seems likely that a homology manifold has a canonical topological normal bundle (see Section 5). In this case we could classify normal bundles of homology manifolds by maps to $(B_{\text{TOP}}) \times \mathbf{Z}$, the $\mathbf{Z}$ factor being $(i(X)-1)/8$. Project to B_G as usual, then the fiber is $(G/\text{TOP}) \times \mathbf{Z}$. One of the wonderful things about G/TOP is that it is nearly periodic; there is a natural equivalence $\Omega^4(G/\text{TOP}) \simeq (G/\text{TOP}) \times \mathbf{Z}$. Including homology manifolds in the picture would give an *exactly* periodic fiber. Exact periodicity in the classifying space would lead to (minor) improvements in the formal properties of surgery theory.

In Section 1 we state the main theorem, and indicate corrections to the statements in [2]. Section 2 explains the error in the original proof, and gives counterexamples to easy repairs. In Section 3 we indicate how the correct parts of the

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