THE ASYMPTOTIC BOUNDARY OF A SURFACE IMBEDDED IN $\mathbf{H}^3$ WITH NONNEGATIVE CURVATURE

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Introduction. From a function-theoretic standpoint, a noncompact complete Riemann surface M with nonnegative curvature has only one point "at infinity." If M is imbedded isometrically in hyperbolic space then one can identify an asymptotic boundary $\partial_\infty M$ as the limit points of M on the ideal boundary of hyperbolic space. We will usually work in the ball model, $\mathbf{B}^3$. The ideal boundary of $\mathbf{H}^3$ is naturally identified with the unit sphere. The asymptotic boundary of M is the set of limit points of M on the unit sphere with respect to the Euclidean topology of $\mathbf{B}^3$. We will prove the following theorem.

THEOREM. *If M is a C^5 complete imbedding of $\mathbf{R}^2$ into $\mathbf{H}^3$ with nonnegative Gauss curvature then the asymptotic boundary of M is a single point.*

The proof uses the hyperbolic Gauss map defined in [4] and draws heavily on results obtained there on surfaces represented as envelopes of horospheres. To apply the machinery of [4] we will prove several propositions on the Gauss map of convex surfaces in $\mathbf{H}^3$ which generalize known results from Euclidean space. By a convex surface M we shall mean a surface which bounds a geodesically convex region D. This is equivalent to the condition that every point of M have a supporting plane, [7, p. 8.10].

It is an easy consequence of Cohn-Vossen's inequality (see [5]),

$$\int_M K\,dA \leq 2\pi\chi,$$

which always holds for complete surfaces with nonnegative curvature, that M is topologically equivalent to a sphere, plane, or cylinder. If K is nonzero at any point then M must be a plane or a sphere. The horospheres are examples of imbeddings of $\mathbf{R}^2$ into $\mathbf{H}^3$ with nonnegative curvature. It is reasonable to inquire if there are any nontrivial examples. In the third section we construct a family of deformations of the horosphere through embedded surfaces with strictly positive curvature.

It would be interesting to know if the hypotheses:

(a) M is complete

(b) M is immersed

(c) M has nonnegative curvature

imply that M is imbedded. If one appends the hypothesis that $\partial_\infty M$ is a single point then it follows that M is imbedded. In fact a stronger result is true.

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