WILD SURFACES HAVE SOME NICE PROPERTIES

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This paper was stimulated by hearing in lectures such remarks as the following: "We consider only smoothable surfaces in 3-manifolds so that we can suppose that their intersections with the 2-skeletons of triangulations are nice." This led me to wonder if we might not make the intersections nice even if the surfaces were not smoothable but wild. See Theorem V.3.

I would prefer to give complete proofs of the results in this paper so that the results would be believed even by mathematical agnostics—those who doubt things for which they do not have complete proofs. Complete proofs are given here for the theorems in the main section (§V) of this paper but the proofs of the preliminary theorems come mostly from the literature. Section V deals with the intersection of wild surfaces in triangulated 3-manifolds with other objects in these 3-manifolds, and the preceding sections deal with theorems related to this treatment.

Throughout this paper we deal with 3-manifolds without boundary. *Surfaces* are 2-manifolds imbedded as closed sets in 3-manifolds. Although there are related results about 3-manifolds with boundaries and surfaces without boundaries, they are not treated here. We do not suppose that manifolds and surfaces are compact. Throughout this paper we use M^3 to denote a triangulated 3-manifold with metric ρ.

I. Pushing tame sets to polyhedral ones. One of the important results of the fifties shows that homeomorphisms of 3-manifolds are not wild. A result [10, Theorem 2; 2, Theorem 9] may be stated as follows. We use M_0^3 to denote a triangulated 3-manifold (perhaps different from M^3).

THEOREM I.1. *Suppose*
U is an open subset of M_0^3,
h is a homeomorphism of U into M^3, and
$\epsilon(x)$ is a positive continuous function defined on U.
Then there is a homeomorphism $g: U \to h(U)$ such that
g is PL *and*
$\rho(g(x), h(x)) < \epsilon(x)$ for each $x \in U$.

At the expense of complicating the statement of Theorem I.1 we could have added that if C is a closed subset of U on which h is locally PL, there is such a g that agrees with h on C.

QUESTION. Suppose U, h are as given in Theorem I.1. Is there an isotopy H_t $(0 \leq t \leq 1)$ of U onto $h(U)$ such that $h = H_0$ and each H_s is polyhedral for $s \in (0, 1]$? It would be especially nice to get an H_t that connects h with a nearby g in

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