SPECTRAL INVARIANTS OF FOLIATIONS

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One of the problems in foliation theory is to relate the transverse geometry of the foliation to its topological invariants, the exotic classes. In this paper we introduce a spectral invariant related to the transverse geometry for an important class of foliations and relate it to exotic characteristic numbers.

A Lie group acting by isometries with constant orbit dimension generates a Riemannian foliation. In this paper we study the case of R^n acting locally freely by isometries, this being an interesting class of foliations; the study of a much larger class of foliations can also be reduced to that of R^n.

Let R^n act by isometries locally freely on a compact oriented $(4k-1)$-manifold M. Let f be a symmetric homogeneous polynomial of degree k in $2k$ indeterminates with integral coefficients. For $\theta \neq 0$ in R^n we construct an eta function $\eta_f(s;\theta)$. Eta is constructed from the infinitesimal generator of the R action corresponding to θ and the transverse signature operator (with coefficients) to the orbits of the R action. We relate the value $\eta_f(s;\theta)$ at $s=0$ to the Simons characteristic number $S_f[M]$ associated to the codimension $4k-n-1$ Riemannian foliation arising from the R^n action and f. We assume throughout this paper that our foliations are oriented.

THEOREM 1. *For generic θ, $\eta_f(s;\theta)$ converges absolutely for* $\mathrm{Re}(s)$ *large and extends to a meromorphic function on the s plane with a finite value at $s=0$. $\eta_f(0;\theta)$ is independent of θ and*

$$\eta_f(0;\theta)=(-1)^k 2^{2k+1} S_f[M] \mod Z[\tfrac{1}{2}].$$

REMARK. Generic is defined in Section 1. Thus $\eta_f(0;\theta)$ is independent of θ for generic θ.

As a corollary to the method of proof we obtain the following.

THEOREM 2. *Let R^n act by isometries on the compact, oriented $4k$ manifold W with boundary M with the action locally free on the boundary. Let η_f be the eta function for the action on M, and let Γ be the fixed set for the action on W. For generic θ,*

$$\eta_f(0;\theta)=(-1)^k 2^{2k+1}\,\mathrm{Residue}(\theta,f,\Gamma) \mod Z.$$

Here residue is that of [6] and [5].

COROLLARY 2. $\eta_f(0;\theta)=0 \mod Z[\frac{1}{2}]$ *when $n>2$ for generic θ.*

REMARK. This allows us to regard $\eta_f(0;\theta)$ as an obstruction to extending an isometric locally free R action to an isometric locally free R^n action for $n>2$.

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