IMMERSIONS EQUIVARIANT FOR A GIVEN KILLING VECTOR II

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0. Introduction. In [1], we showed that any complete Riemannian manifold with a 1-parameter subgroup of isometries and sectional curvatures bounded above by $-c<0$ cannot be immersed isometrically and equivariantly into any Euclidean space.

On the other hand, we have a negatively curved complete revolution surface whose order of the Gauss curvature at infinity is (distance from a fixed point)$^{-2-e}$, where e is positive.

In this paper, we know that the above estimate is best. That is, we have the following.

THEOREM A. *Let M be a complete Riemannian manifold of negative curvature and ρ a 1-parameter subgroup of isometries acting nontrivially on M. If there exists a point $x \in M$ such that the maximum of sectional curvatures on the geodesic ball of radius s with center x is bounded above by $-As^{-2}$, $A>0$ for large s, then M does not admit any ρ-equivariant isometric immersion into Euclidean spaces.*

Furthermore, we give analogous results to [1] in the case that the ambient space is a hyperbolic space. That is, we obtain the following.

THEOREM B. *Let M be a complete Riemannian manifold, and let $\rho(\theta)$ $(\theta \in \mathbf{R})$ be a 1-parameter subgroup of isometries acting nontrivially on M. If the sectional curvatures of $M \leq -c < -1$, then M has no ρ-equivariant isometric immersion into any hyperbolic space with sectional curvature -1.*

THEOREM C. *Let M be an n-dimensional non-compact type symmetric space with Ricci curvature $-(n-1)c$, $c>1$, and let ρ be a 1-parameter subgroup of isometries acting nontrivially on M. Then M has no ρ-equivariant isometric immersion into any hyperbolic space with sectional curvature -1.*

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1. Revolution surfaces with negative curvatures in $\mathbf{R}^3$. In this section, we study "the order of the Gauss curvature at infinity" of some complete revolution surfaces with negative curvature in $\mathbf{R}^3$.

Let (r,θ) be the polar coordinate of $\mathbf{R}^2$ and (t,r,θ) the coordinate of $\mathbf{R}^3$. We give a revolution surface S by

$$\mathbf{R}\times S^1 \ni (t,\theta) \to (t,\ \tau(t)\cos\theta,\ \tau(t)\sin\theta) \in \mathbf{R}^3,$$

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