HARDY SPACES AND BMO-FUNCTIONS INDUCED BY ERGODIC FLOWS

Jun-Ichi Tanaka

1. Introduction. Let X be a measure space with probability measure m, and let $\{T_t\}_{t\in\mathbf{R}}$ be an ergodic measurable action of the real line $\mathbf{R}$ on X preserving m. The *ergodic Hilbert transform* on X, $H_X\phi$, of a function ϕ in $L^1(X)$ is defined by the formula:

$$(1.1)\qquad (H_X\phi)(x)=\lim_{\epsilon\to+0}\frac{1}{\pi}\int_{\epsilon<|t|<1/\epsilon}\phi(T_{-t}x)\frac{dt}{t}$$

for a.e. x in X. The existence of this limit is shown in [2]. Let $H^\infty(X)$ be the subalgebra of $L^\infty(X)$ consisting of functions of the form $\phi+iH_X\phi$, and let $H_0^\infty(X)$ be the subspace of all functions in $H^\infty(X)$ with mean value zero. The space $H^p(X)$ (resp. $H_0^p(X)$), $0<p<\infty$, is defined to be the closure of $H^\infty(X)$ (resp. $H_0^\infty(X)$) in $L^p(X)$. The measure m is multiplicative on $H^\infty(X)$, and $H^\infty(X)$ becomes a weak*-Dirichlet algebra in $L^\infty(X)$ (cf. [10], [16], and Proposition 5.1 in Section 5).

Let Y be a measure space with probability measure m_1, and let T be an ergodic measure preserving transformation on Y. Suppose that F is a bounded measurable function on Y, bounded away from zero, and normalized to have integral one. Throughout this paper, we shall always assume that the ergodic flow $(X,\{T_t\}_{t\in\mathbf{R}},m)$ is the "special flow under the function F" generated by ergodic dynamical system (Y,T,m_1). More precisely, we define τ to be the function by the formula

$$(1.2)\qquad \tau(y,n)=\begin{cases}\sum_{j=0}^{n-1}F(T^jy) & \text{if } n>0,\\ 0 & \text{if } n=0,\\ -\tau(T^ny,-j) & \text{if } n<0,\end{cases}$$

for each integer n and each y in Y. Let X be the region of $Y\times\mathbf{R}$ under the graph of F, that is,

$$X=\{(y,s):y\in Y \text{ and } 0\le s<F(y)\},$$

and let m be the restriction of $dm_1\times dt$ to X. Then it is easy to see that m is a probability measure on X by the hypotheses of F. By using (1.2), a measure preserving transformation group $\{T_t\}_{t\in\mathbf{R}}$ on X is defined by the formula

$$(1.3)\qquad T_t(y,s)=(T^ny,s+t-\tau(y,n))$$

Received March 23, 1984. Final revision received August 11, 1984.

This research was partially supported by a Grant-in-Aid for Scientific Research (No. 57740097) from the Japanese Ministry of Education.

Michigan Math. J. 32 (1985).