DOUBLY SLICED KNOTS AND DOUBLED DISK KNOTS

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A *doubly sliced knot* is, roughly speaking, a knot which can be realized as a slice of a trivial knot. This notion was introduced by Fox [4] and Sumners [16]; Kearton [9] and Stoltzfus [18] have extracted algebraic obstructions from the middle dimensional homology of the infinite cyclic covering of the complement of the knot. In the case of higher dimensional simple knots the vanishing of this obstruction is necessary and sufficient for a knot to be doubly sliced ([16], [7], [19]). One can, in addition, ask whether there are further obstructions in the case of non-simple knots. In comparison, recall that there are no such further obstructions to a knot being sliced [12].

In his recent Ph.D. dissertation [15], D. Ruberman has used Casson-Gordon type invariants to define such obstructions and construct "algebraically" doubly sliced knots (i.e., satisfying the Sumners or Stoltzfus conditions) of every dimension which are not doubly sliced (also see [5]).

In the first part of this note we present a simpler approach by showing that the entire cohomology ring of the infinite cyclic covering of the complement of a knot represents a generalization of the Sumners and Stoltzfus obstructions. Examples in dimension 2 and 4 show this is a non-trivial generalization, but I have not yet found examples in higher dimensions.

The analogy between doubly sliced knots and codimension one submanifolds of Euclidean space, which is pointed out in [5] and [15], is also apparent in this result—compare [14].

We illustrate the usefulness of this approach by some examples: (1) the 2-twist spin of any 2-bridge knot and (2) the knots constructed by Cappell-Shaneson [3] are all shown to be not doubly sliced.

In the second part we show that a large number of doubly sliced knots can be generated by the process of "doubling" a disk knot. This generalizes the observation of Sumners [16] that the connected sum of any knot with its inverse is doubly sliced and, furthermore, includes all spun and super-spun knots [2]. It is not hard to find doubly sliced knots, in low dimensions, which are not doubled disk knots, but I have not been able to find higher-dimensional examples.

We also discuss, following suggestions to the author by D. Sumners, the related phenomenon of invertible disk knots (see [16]). The above result about doubled disk knots follow from the fact that "suspensions" of disk knots are invertible. On the other hand, examples of invertible knots which are not suspensions can be found in every dimension using the construction of [6].

However, I have found no examples which are "1-simple", i.e. the complements of the disk knot and its boundary knot have abelian fundamental group.

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