ON TWO NOTIONS OF THE LOCAL SPECTRUM FOR SEVERAL COMMUTING OPERATORS

Jörg Eschmeier

S. Frunză and E. Albrecht initiated the study of spectral decompositions for finite systems $a=(a_1,\dots,a_N)$ of commuting operators on a Banach space X. S. Frunză used the results of J. L. Taylor [8; 9] concerning the spectrum and the analytic functional calculus to develop his concept of decomposable N-tuples. He proved in [6] that a spectral capacity E of an N-tuple a is necessarily of the form

$$E(F)=\{x\in X;\, \sigma_a(x)\subset F\}, \tag{1}$$

where $\sigma_a(x)$ is the so-called local spectrum of a in x, i.e. the complement of the union of all open sets $U\subset \mathbf{C}^N$ on which there is a solution of

$$xs_1\wedge\cdots\wedge s_N=(\bar\partial\oplus\alpha)\psi.$$

Earlier E. Albrecht had suggested another definition of the local spectrum, now called the local analytic spectrum $\gamma_a(x)$ of a in x. In [1] he defined $\gamma_a(x)$ to be the complement of the union of all open sets $U\subset \mathbf{C}^N$ on which there are analytic functions $f_i\colon U\to X$ satisfying

$$x=\sum_{i=1}^{N}(z_i-a_i)f_i(z),\quad z\in U.$$

S. Frunză has shown in [6] that $\sigma_a(x)\subset\gamma_a(x)$, that equality holds for decomposable N-tuples and in [7] that $\gamma_a(x)$ is contained in the spectral hull of $\sigma_a(x)$.

The aim of this paper is to prove that $\sigma_a(x)=\gamma_a(x)$ holds for each $x\in X$. This enables us to give a very simple proof of Equation (1) for decomposable N-tuples.

Equality of the local spectra. Let X be a complex Banach space and $a=(a_1,\dots,a_N)$ a system of commuting continuous linear operators on X. For an open set $U\subset\mathbf{C}^N$ we denote by $\mathcal{O}(U,X)$ the space of all analytic functions and by $C^\infty(U,X)$ the space of all C^∞-functions on U with values in X. If B is a K-module let $A_p(\mathbf{C}^N,B)$ be the K-module of all forms with degree p over $\mathbf{C}^N$ and coefficients in B. Special 1-forms needed are

$$\alpha(z)=(z_1-a_1)s_1+\cdots+(z_N-a_N)s_N,\qquad \alpha=(z_1-a_1)s_1+\cdots+(z_N-a_N)s_N,$$

$$\bar\partial=(\partial/\partial\bar z_1)d\bar z_1+\cdots+(\partial/\partial\bar z_N)d\bar z_N\quad\text{and}$$

$$\bar\partial\oplus\alpha=(\partial/\partial\bar z_1)d\bar z_1+\cdots+(\partial/\partial\bar z_N)d\bar z_N+(z_1-a_1)s_1+\cdots+(z_N-a_N)s_N.$$

Received April 6, 1983.
Michigan Math. J. 30 (1983).