ON TUBULAR NEIGHBORHOODS OF FIXED POINTS OF LOCALLY SMOOTH ACTIONS

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1. Introduction. G. E. Bredon, in his book [1], defines a locally smooth action of a compact Lie group on a topological space. Locally smooth actions of compact Lie groups on manifolds form a class of actions lying between that of topological actions and that of smooth actions.

An action of a compact Lie group G on a topological space M is said to be locally smooth if there exists a linear tube about each orbit. A linear tube is defined as follows: Let H be a closed subgroup of G and $G(x)=\{g\cdot x \mid g\in G\}$ be an orbit of type G/H, and let V be a Euclidean space on which H acts orthogonally. Then a linear tube about $G(x)$ in M is a G-equivariant imbedding $\phi: G\times_H V \to M$ such that $\phi(G\times_H V)$ is an open neighborhood of $G(x)$ in M, where $G\times_H V$ is the twisted product of G and V. The twisted product is defined to be the orbit space of the action H on $G\times V$ given by $h\cdot(g,v)=(g\cdot h^{-1}, h\cdot v)$ for $h\in H$, and $(g,v)\in G\times V$. If G acts locally smoothly on M, then M is a topological manifold and any connected component of the fixed point set $F(G,M)=\{x\in M \mid g\cdot x=x$ for all $g\in G\}$ is a topological submanifold of M. It is well known that there is a linear tube about each orbit of any smooth action. Thus a smooth action of a compact Lie group is locally smooth [1]. The following facts about locally smooth actions are shown in [1: pp. 179–185].

If a compact Lie group G acts locally smoothly on the m-manifold M with the orbit space M/G connected, then there exists a maximum orbit type G/H for G in M, i.e., H is conjugate to a subgroup of the isotropy group $G_x=\{g\in G \mid g\cdot x=x\}$ of G at each $x\in M$. The orbits of this type are called the principal orbits. The union $M_{(H)}$ of the orbits of maximum orbit type G/H is open and dense in M and its image $M_{(H)}/G$ in M/G is also connected. Let V be a linear slice at x ($x\in V\subset M$, $G_x(V)=V$). Then the orbit $G(x)$ is principal if and only if G_x acts trivially on V and $G\times_{G_x} V=(G/G_x)\times V$. Let $G(x)$ be an exceptional orbit (i.e., $\dim G(x)=\dim$ of a principal orbit, but they are not equivalent), and let V be a linear slice at the point x. If $H\subset G_x$ is a principal isotropy group for G_x on V, then H is just the ineffective part of G_x on V. Therefore G_x/H is a finite group acting effectively on the slice V. If $F(G_x, V)$ has codimension one in V, i.e., $G(x)$ is a special exceptional orbit, then G_x/H has order two and acts by reflection across the hyperplane $F(G_x, V)$ of V.

The purpose of this paper is to study an invariant tubular neighborhood of the fixed point set $F(G,M)$ of a locally smooth action of a compact Lie group G on a manifold M. An open (or closed) invariant tubular neighborhood of $F(G,M)$ in M is a normal bundle $p: E\to F(G,M)$ such that E is a G-invariant open (or closed) neighborhood of $F(G,M)$ in M, and the action of G on each fiber is

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