EXAMPLES OF CELL-LIKE MAPS THAT ARE NOT SHAPE EQUIVALENCES

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1. Introduction. For the most part, cell-like maps behave in the expected fashion by being hereditary homotopy equivalences in the setting of absolute neighborhood retracts and by being hereditary shape equivalences in the general setting of metric spaces; see [7], [9], [11], [14], [15]. An exception is an example due to J. Taylor of a cell-like map F from a compactum T with nontrivial shape onto the Hilbert cube Q [16]. In this paper, we verify a suspicion that a careful analysis of this example would be beneficial by producing examples of:

(1) a cell-like map H from the compactum T onto Q such that the nondegeneracy set $N_H = \{q \in Q : H^{-1}(q) \neq \text{point}\}$ is a countable union of finite dimensional compacta;

(2) a map from a compactum onto Q whose point-inverses are finite dimensional absolute retracts and which is not a shape equivalence; and

(3) a locally contractible compactum Z which is not an ANR, compacta X and Y which are ANR's, and cell-like maps $g: X \to Z$ and $f: Z \to Y$ with the property that $N_g \cap f^{-1}(N_f) = \varnothing$.

A central feature is an analysis which includes a verification that the map F originally constructed by J. Taylor is a hereditary shape equivalence over its nondegeneracy set N_F and, of course, is a homeomorphism over $Q - N_F$.

A by-product of the techniques used to establish (3) is that if the compactum $T \subset Q$ is embedded as a Z-set, then the adjunction space $Q \cup_F Q$ is locally contractible. Moreover, the analysis in Section 7 produces a basis for $Q \cup_F Q$ consisting of contractible open sets. Since $Q \cup_F Q$ is not an ANR, as the induced map $\tilde{F}: Q \to Q \cup_F Q$ is cell-like but is not a hereditary shape equivalence ([9; Corollary 2]), Question (ANR 1) in [6] has a negative answer.

2. Preliminaries. In order to facilitate coping with the abundance of notation appearing as we simultaneously manipulate up to three inverse sequences, we adopt the following conventions, which will be adhered to scrupulously throughout the paper. The maps in an inverse sequence are denoted by lower case Greek letters. The inverse limit of an inverse sequence $\{X_n, \alpha_n\}$ is written $(X_n)_\infty$ and the compositions and induced maps are denoted, respectively, by $\alpha_{ij}: X_i \to X_j$ for $i \geqslant j$ (where $\alpha_{ii} = \text{identity}$ and $\alpha_{i,i-1} = \alpha_i$) and $\alpha_{\infty i}: (X_n)_\infty \to X_i$. Maps between inverse sequences are denoted by lower case Roman letters while the induced map between the limits is denoted by the capital of the same letter; for example, $\{g_n\}: \{X_n, \alpha_n\} \to \{Y_n, \beta_n\}$ and $G: (X_n)_\infty \to (Y_n)_\infty$.

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