QUASI-INVARIANT MEASURES AND MAXIMAL ALGEBRAS ON MINIMAL FLOWS

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Dedicated to Professor Takizo Minagawa on his 70th birthday.

1. Introduction. Let Γ be a dense subgroup of the real line $\mathbf{R}$, endowed with the discrete topology, and let K be the dual group of Γ. For each t in $\mathbf{R}$, e_t denotes the element of K defined by $e_t(\lambda)=e^{i\lambda t}$ for any λ in Γ. The mapping from t to e_t embeds $\mathbf{R}$ continuously into K, and it is well known that the translation by e_t defines a strictly ergodic flow. Fix a positive γ in Γ, and let K_γ be the compact subgroup consisting of all x such that $x(\gamma)=1$. If we put $\beta(x,t)=x+e_t$, then β carries $K_\gamma x[0,2\pi/\gamma)$ continuously onto K. Furthermore, β is one to one and its inverse, β^{-1}, is continuous except at points on K_γ. It follows that Borel sets are taken to Borel sets in both directions. Thus K is represented measure theoretically, and almost topologically, as a product space $K_\gamma x[0,2\pi/\gamma)$. Also it can be easily seen that the above flow, $x+e_t$, on K can be characterized by the homeomorphism S on K_γ defined by $S(y)=y+e_{2\pi/\gamma}$. This local product decomposition is very useful for understanding the structure of K, and is also highly important in the study of analyticity on compact abelian groups (cf. [6; Chapter II], and [5, Chapter VII, Section 6]). Especially, we notice that, by using this decomposition, a representation of quasi-invariant measures on K was shown by deLeeuw and Glicksberg [2].

Our principal objective in this article is to extend the local product decomposition in quotients of the Bohr group to minimal flows, and particular attention is given to representing quasi-invariant measures on minimal flows. Moreover, as an application of this representation, we investigate the maximality of algebras of analytic functions associated with a minimal flow. Conceivably, our proof enables us to make clearer the relation between Forelli's generalization [4] of Wermer's maximality theorem and Muhly's result [7; Corollary 3.1] concerning maximal weak-* Dirichlet algebras.

On the other hand, a famous theorem of Ambrose [1] showed that any measurable ergodic flow can be represented as a flow built under a function. Our main result may be regarded as a refinement of this theorem concerning continuous flows.

In the next section, we present some preliminary material which we shall need. In Section 3, our representation of a minimal flow, Theorem 3.3, is obtained, and we also give a representation of quasi-invariant measures. We deal with analytic measures and provide simpler proofs of two known theorems concerning maximal algebras in Section 4. We close with some remarks in Section 5.

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