# A COVARIANT VERSION OF Ext

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**1. Introduction.** If $H$ is a separable Hilbert space, and $G$ is a topological group with a strongly continuous unitary representation on $H$, then $G$ acts by conjugation on the bounded linear operators $L(H)$ and on the Calkin algebra $Q(H) = L(H)/K(H)$, where $K(H)$ denotes the compact operators. If $G$ also acts on a $C^*$-algebra $A$, then $*$-monomorphisms of $A$ into $Q(H)$ which are compatible with these two actions are called covariant extensions. In this paper we construct three groups out of equivalence classes of covariant extensions, when $G$ is 2nd countable and compact, and $A$ is separable and nuclear. Two of these groups are the analogues of the weak and strong Ext groups of [3].

A systematic study of covariant extensions and of several closely related topics has been undertaken in [4], [8], and [11]. In [4] an equivalence relation was defined on the set of covariant extensions and in the case when $G$ is finite and $A = C(X)$, it was proved that the equivalence classes of covariant extensions together with a binary operation induced by direct sum forms an abelian group. We have been able to generalize this result somewhat, but at the expense of a slightly weaker notion of equivalence. However, we shall show that for $G$ finite the two notions of equivalence coincide.

Section 2 contains some preliminary definitions together with a covariant version of Stinespring's Theorem [12]. In Section 3 we introduce three equivalence relations on the covariant extensions. Following the ideas of [2], we combine the covariant version of Stinespring's Theorem together with a result on the existence of covariant completely positive liftings to prove that, for each of the three equivalence relations, the collection of equivalence classes forms a group. We close Section 3 by describing some of the relationships between these three groups. Some discussion of the possibility of extending these results to noncompact $G$ is included. In Section 4 we calculate each of these groups for the case of the circle group acting on itself by multiplication.

Our techniques and constructions are similar to those of Kasparov ([5], [6], and [7]), but the special case we are interested in, while considerably simpler, allows for some additional structure. In particular, our theory admits a natural action of the representation ring of $G$, while this does not appear to be possible in Kasparov's theory. It is this action that facilitates the calculations of Section 4. In addition, we feel that our construction is more amenable to generalizations to non-compact $G$. We discuss the relationships between our theory and Kasparov's in Section 5.

We would like to acknowledge many helpful conversations with William Paschke. In addition, this paper rests heavily on the ideas introduced by Kaminker, Loebl, and Schochet in their sequence of papers.

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Received July 9, 1980. Revision received February 18, 1981.
Research supported in part by a grant from the NSF.
Michigan Math. J. 29 (1982).