ZEROS OF PARTIAL SUMS OF LAURENT SERIES

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To the memory of David L. Williams.

Introduction. The present investigation was prompted by a theorem and a conjecture of Abian. Before stating Abian's problem, I introduce some general assumptions and notations.

Consider a Laurent series

$$\sum_{j=-\infty}^{+\infty} a_j z^j = f(z) \quad (z = re^{i\theta}), \tag{1}$$

whose exact annulus of convergence is

$$\rho < |z| < R \quad (0 \le \rho < R \le +\infty). \tag{2}$$

The *partial sums* of (1):

$$T_n(z) = \sum_{j=-p}^{n} a_j z^j \quad (p = p(n),\ n = 1, 2, 3, \ldots), \tag{3}$$

are defined in terms of a given sequence $\{p(n)\}_{n=1}^{\infty}$ of positive integers. There are no restrictions on $p(n)$ other than

$$\lim_{n \to +\infty} p(n) = +\infty. \tag{4}$$

The sums $T_n(z)$ are also called *sections* or *truncations* of the Laurent series.

I find it convenient to impose, throughout the paper, one additional assumption:

A. *For some* $\xi > 0$, *the compact annulus*

$$\mathfrak{A} = \{z : e^{-\xi} \le |z| \le e^{\xi}\} \quad (\rho < e^{-\xi},\ e^{\xi} < R) \tag{5}$$

contains no zeros of $f(z)$.

It is clear that the condition A has the character of a normalization; it will not affect the generality of my results. Since the zeros of $f(z)$ play a dominant role in this note, it is natural to consider, beside (1), the factored form

$$f(z) = Cz^l \psi(1/z) \varphi(z), \tag{6}$$

where $C \neq 0$ is a constant, l is an integer (not necessarily positive), $\varphi(z)$ and $\psi(z)$ are analytic functions represented by

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