ORTHOGONAL POLYNOMIALS ASSOCIATED WITH AN INFINITE INTERVAL

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SECTION 1

1.1. *Introduction.* A problem of great interest in approximation theory is the asymptotic behavior of the largest zero of orthogonal polynomials. What we show in this paper, in Theorem 1, is that when such knowledge is available, we can then get further information about the asymptotic behavior of the norms of the orthogonal polynomials, assumed to be monic, and in some instances, we also get the asymptotic distribution of all of the zeros of the orthogonal polynomials. Since the largest zero behavior for the class of weight functions

$$W_{p,m}(x) = |x|^p \exp(-|x|^m), \quad p > -1, \quad m > 0$$

has been determined for $m = 2, 4, 6$, p arbitrary, in [3] and [4], Theorem 1 can be applied to these cases, formulated as Theorem 2, and we are led to the explicit determination of the zero distributions.

Thus, aside from the intrinsic interest in this class of weights, Theorem 1 provides a general method for converting largest zero asymptotics to the asymptotic behavior of other important parameters of orthogonal polynomials.

Added in revision. After this paper was submitted Professor Paul Nevai informed me of an alternate approach to this problem which will appear in "On Asymptotic Average Properties of Zeros of Orthogonal Polynomials," by Paul G. Nevai and Jesus S. Dehesa.

1.2. Let $W(x)$ be a non-negative continuous function defined for $x \in R = (-\infty,\infty)$ and satisfying $0 < \int |t|^n W(t)\,dt < \infty$, $n = 0, 1, \ldots$. Such a function is called an admissible weight function. There are unique polynomials, $p_n(x) = \gamma_n X^n + \ldots$, $n = 0, 1, \ldots$, which satisfy

$$\int p_m(x) p_n(x)\, W(x)\, dx = \delta_{m,n}, \quad n, m = 0, 1, \ldots.$$

These are the orthonormal polynomials associated with the weight function $W(x)$. Let $N_n(W) = 1/\gamma_n$, and refer to $N_n(W)$ as the norm of the monic orthogonal polynomial $P_n(x) = p_n(x)/\gamma_n$. We assume that $W(x)$ is positive for values of x

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