SOME REPRESENTING MEASURES FOR THE BALL ALGEBRA

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In this paper, M_0 denotes the class of those (Borel) probability measures ρ on the sphere S (the boundary of the open unit ball B in $\mathbf{C}^n$) that satisfy

$$\int_S f\,d\rho = f(0) \tag{1}$$

for every f in the ball algebra $A(B)$. [Recall that $f \in A(B)$ if and only if f is a continuous complex function on $\bar{B}$ and f is holomorphic in B. The members of M_0 "represent" the homomorphism $f \to f(0)$ of $A(B)$ onto $\mathbf{C}$.]

When $n = 1$, M_0 has exactly one member, namely normalized Lebesgue measure on the unit circle T. In general, M_0 is convex and weak*-compact, but it turns out to be a very large set when $n > 1$.

The "obvious" members of M_0 are the *circular* probability measures μ on S. By definition, these satisfy

$$\int_S v(e^{i\theta}\zeta)\,d\mu(\zeta) = \int_S v\,d\mu \tag{2}$$

for every $v \in C(S)$ and for every real θ. Indeed, if (2) holds and $f \in A(B)$, then $\lambda \to f(\lambda\zeta)$ is in the disc algebra $A(U)$ ($U = B^1$), so that

$$\int_S f\,d\mu = \int_S d\mu(\zeta)\,\frac{1}{2\pi}\int_{-\pi}^{\pi} f(e^{i\theta}\zeta)\,d\theta = f(0), \tag{3}$$

by Fubini's theorem.

To see some others, take $n = 2$, for simplicity. Let τ be *any* probability measure on $\bar{U} \subset \mathbf{C}$ that satisfies

$$\int_{\bar{U}} g\,d\tau = g(0) \tag{4}$$

for every $g \in A(U)$. For example, τ might be concentrated on a simple closed curve Γ in U that surrounds the origin, in such a way that τ solves the Dirichlet problem at 0 relative to the domain bounded by Γ. The measure ρ that satisfies

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