A RUDIN-CARLESON THEOREM FOR UNIFORMLY CONVERGENT TAYLOR SERIES

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Let T be the unit circle in the complex plane and let m be normalized Lebesgue measure on T. For a continuous complex-valued function $f(z)$ on T and an integer j, define the Fourier coefficient $\hat{f}(j)$ by $\hat{f}(j) = \int_T f(z) z^{-j} dm(z)$. The letter A will stand for the space of continuous functions $f(z)$ on T such that $\hat{f}(j) = 0$ for $j < 0$, while U will denote the set of all $f \in A$ such that

$$f(z) = \lim_{n\to\infty} \sum_{j=0}^{n} \hat{f}(j) z^j$$

uniformly on T. Recall the classical Rudin-Carleson theorem.

THEOREM 1. ([6], [2]). *Let $K \subseteq T$ be a closed set with $m(K) = 0$ and suppose that g is a continuous function on K. There exists $f \in A$ such that $f(z) = g(z)$ for $z \in K$ and $|f(z)| < \sup\{|g(w)|: w \in K\}$ if $z \notin K$.*

The purpose of this note is to strengthen Theorem 1 as follows:

THEOREM 2. *Let K and g be as in Theorem 1. Then there exists $f \in U$ such that the conclusions of Theorem 1 are valid.*

This theorem answers a question on p. 89 of [5] and extends certain previously known results. (See, e.g., [4].)

We now begin the proof. Let $D_n(z) = \sum_{j=0}^{n} z^j$ so that $\sum_{j=0}^{n} \hat{f}(j) z^j$ is equal to the convolution over the group T $D_n * f(z)$. Let Y be the set $\{0\} \cup \{n^{-1}\}_{n=1}^{\infty}$, and let $\tilde{T}$ be the space $T \times Y$. Then, if $f \in A$, $f \in U$ if and only if the function

$$\tilde{f}(z,y) = \begin{cases} f(z) & \text{if} \quad y = 0 \\ D_n * f(z) & \text{if} \quad y = n^{-1} \end{cases}$$

is continuous on $\tilde{T}$. Thus U corresponds to a uniformly closed subspace $\tilde{U}$ of the space of continuous functions on $\tilde{T}$. The conclusion of our theorem can be stated as follows: if $K \subseteq T$ is compact and of measure zero, then $\tilde{K} = \{(k,0) : k \in K\}$ is a set of interpolation for the space $\tilde{U}$ of functions on $\tilde{T}$. The generalized Rudin-Carleson theorem [1] now shows that it is enough to establish the following fact.

Received October 23, 1978. Revision received November 2, 1978.
Partially supported by NSF Grant MCS 76-02267-A01.

Michigan Math. J. 27 (1980).