SOME QUOTIENT SURFACES ARE SMOOTH

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Let $G\backslash\mathbf{V}$ be the quotient variety where G is a reductive group acting linearly on a vector variety $\mathbf{V}$. Then $G\backslash\mathbf{V}$ is an affine variety, whose regular functions may be identified with the regular functions on $\mathbf{V}$, which are invariant under the action of G.

It has become increasingly clear how very special the singularities of $G\backslash\mathbf{V}$ actually are [1,4,5 and 6]. In this note, we will show that, if G is a connected semi-simple group and $G\backslash\mathbf{V}$ is two dimensional, then $G\backslash\mathbf{V}$ must be non-singular. This result was conjectured by V. L. Popov, and it was brought to my attention by V. Kać.

A key step in my proof is the application of Mumford's smoothness criterion in terms of the local fundamental group. This idea was suggested by Mumford himself. The first step in the proof will be to check directly that the algebraic fundamental group of a large open subvariety of the quotient $G\backslash\mathbf{V}$ is trivial.

1. ETALE COVERINGS OF QUOTIENT VARIETIES

Let G be a reductive group over an algebraically closed field k of characteristic zero. We will denote the connected component of the identity of G by G_0. We will be working in the category of k-schemes of finite type.

Let X be an affine scheme with a given (morphic) action of the group G. Let $G\backslash X$ denote the quotient variety. Then we have the quotient morphism $\pi: X \to G\backslash X$. Let $f: S \to G\backslash X$ be a morphism. Form the Cartesian square,

$$\begin{array}{ccc} X_S & \rightarrow & X \\ \pi_S \downarrow & & \downarrow \pi \\ S & \xrightarrow{f} & G\backslash X. \end{array}$$

Then, G acts naturally on X_S through its action on X so that S may be regarded as the quotient $G\backslash X_S$ [10].

Next we will study the connected components of X_S.

LEMMA 1.1. *Assume that S is connected. Then,*

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