A GEOMETRIC CONDITION WHICH IMPLIES BMOA

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1. INTRODUCTION

The space BMOA is the collection of analytic functions on the unit disc D which are in the Hardy space H^1 and whose boundary values belong to the space BMO of John and Nirenberg [6].

Recently, Hayman and Pommerenke [5] discovered a geometric characterization of all regions Ω with the property that an analytic function with values in Ω will belong to BMOA. Their characterization uses logarithmic capacities.

At about the same time I independently discovered the sufficiency result along with several applications and generalizations to known results. These applications are given below along with the best norm result which involves a property of logarithmic capacity which may be of independent interest.

2. STATEMENT OF THE RESULTS

The geometric characterization given in [5] is that there exist an $r > 0$ and $\delta > 0$ such that $\operatorname{Cap}(D(w,r)\setminus\Omega) \geq \delta$ for all w in Ω. Here $D(w,r)$ is the closed disc of radius r centered at w and "Cap" denotes the logarithmic capacity of a set.

For a region Ω (Ω is an open, connected subset of $\mathbf{C}$) let

$$\phi(r) = \inf_{w\in\Omega} \frac{\operatorname{Cap}(D(w,r)\setminus\Omega)}{\operatorname{Cap}(D(w,r))}$$

so that $0 \leq \phi \leq 1$. We could replace the denominator with r since the capacity of a disc is its radius. If in the definition of ϕ we replace Cap with a measure, then the condition $\phi(r_0) = \delta > 0$ would not imply a stronger result for large r, i.e., the ratio could remain constant. Surprisingly, the situation with capacities is quite different.

THEOREM 1. *For a region Ω, $\lim_{r\to\infty} \phi(r) = 1$ provided that $\phi(r) \neq 0$ for some $r > 0$. In addition, there exists an $r > 0$ with $2^{-5} \leq \phi(r) \leq 2^{-1/5}$.*

The next is a refinement of that given in [5].

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