BORDISM OF METACYCLIC GROUP ACTIONS

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In this paper the term *manifold* means a differentiable, compact manifold having a unitary (stably almost complex) structure [9]. A *G-action* on such a manifold is a differentiable action of a finite group G preserving the unitary structure. We write Ω_*^U for the bordism ring of closed unitary manifolds; Milnor [9] proved that Ω_*^U is an integral polynomial ring with one generator in each even dimension. Let $\Omega_*^U(G)$ be the bordism of G-actions, as studied, for example, by Stong [13].

The following question, once under active study, has been dormant for several years. Is $\Omega_*^U(G)$ always a free Ω_*^U-module on even-dimensional generators? Stong [13] proved that this is true for abelian p-groups G, and Ossa [10] showed how to extend Stong's result to all abelian groups. Lazarov [8] showed that the answer is also yes if G is a group of order pq for distinct primes p and q. In this paper, we give an affirmative answer for a well-known class of metacyclic groups.

THEOREM. *Suppose all Sylow subgroups of G are cyclic. Then $\Omega_*^U(G)$ is a free Ω_*^U-module on even-dimensional generators.*

Some readers will recall that Landweber and Lazarov have announced such a theorem [7], although they required an additional hypothesis on the group G. Professor Lazarov was kind enough to send me, several years ago, a copy of a manuscript proving the theorem for groups of order $p^m q^n$. The proof given here is very different; although following the general outline proposed in [8], it uses the methods of [11] to reduce the necessary calculations by at least an order of magnitude.

There are six parts to the proof. The first two list some well-known facts we shall require. Part 3 is an outline of the proof. Part 4 recalls the machinery of [11]. The last two parts contain the computations.

1. GROUP THEORY

Let $\mathscr{M}$ be the class of finite groups G such that every Sylow subgroup of G is cyclic. It is clear that if $G \in \mathscr{M}$ then each subgroup and each factor group of G is also in $\mathscr{M}$. By a well-known theorem, (see [5, pp. 146–148], for example), if $G \in \mathscr{M}$ then G may be written as an extension

$$1 \to H \to G \to K \to 1,$$

so that H and K are cyclic and have relatively prime orders. It follows that if n divides the order of G then G possesses a subgroup of order n. We will need the following refinements of this observation.

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