FIXED POINTS OF AUTOMORPHISMS OF LINEAR GROUPS

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INTRODUCTION

In the author's paper, *Algebraic Automorphisms of Algebraic Groups with Stable Maximal Tori,* a counterexample due to D. Winter was given, showing the existence of a solvable linear group in characteristic 2, with automorphism σ, which has two σ-stable maximal tori not conjugate by a σ-fixed point.

This paper generalizes that group for any characteristic $p > 0$. We define first an upper triangular group $\mathfrak{G}$ in $GL(p(p+1), \kappa)$ consisting of p diagonal block matrices, each block being upper triangular in $GL(p+1, \kappa)$. We then define a rational representation θ on $\mathfrak{G}_u$, the unipotent part of $\mathfrak{G}$:

$$\theta : (\mathfrak{G}_u, \cdot) \to (\kappa, +).$$

Our desired group is $G = T \cdot U$ where T is the diagonal maximal torus of $\mathfrak{G}$ and U is the kernel of θ. The automorphism σ of G cyclically permutes the p blocks of a matrix; that is, σ replaces the first block by the second, the second block by the third, etc., and the p^{th} block by the first. Having been previously defined on $\mathfrak{G}$, σ is used in the construction of θ.

PART I

Let $M_i \subseteq GL(p+1, \kappa)$ be upper triangular matrices, for $i = 1, \ldots, p$; κ an algebraically closed field with char $\kappa = p > 0$. Let M be the matrix in $GL(p(p+1), \kappa)$ with $M_1, \ldots, M_p$ along the diagonal, and zeroes elsewhere:

$$M = \begin{bmatrix} M_1 & & & & & \\ & M_2 & & & & \\ & & \cdot & & 0 & \\ & & & \cdot & & \\ & 0 & & & \cdot & \\ & & & & & M_p \end{bmatrix}$$

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