NEARLY MAXIMAL REPRESENTATIONS FOR THE SPECIAL LINEAR GROUP

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1. INTRODUCTION

Since early in this century there has been a continuing interest in the following problem: For a given finite group, G, what are the maximal subgroups of G? This problem is of course most interesting when a family of groups is considered, and examples of such work are the results of Mitchell on $PSL_3(q)$, $PSU_3(q)$ and $PSp_4(q)$, q odd (see [7] and [8] resp.) and those of Hartley for $PSL_3(q)$, q even (see [3]). More recently there is the work of Mwene (see [9]). The problem of finding all the maximal subgroups of $PSL_n(q)$, or of any of the classical groups, is in general not a realistic one, since this amounts to essentially finding all irreducible subgroups of these groups (on their standard modules). A variation on this theme is the following: suppose G is a group, and H is embedded in some known way in G, what are the subgroups of G which contain H? In particular, is H maximal? Burgoyne, Greiss and Lyons [1] considered this problem for G a group of Lie type and H the fixed points of certain automorphisms of G of prime order. In [2], E. Halberstadt considers $\Sigma(X)$, the symmetric group on a finite set X, and its action on $X^{(k)}$, the k-element subsets of X, and shows that the embedding in $\Sigma(X^{(k)})$ or $A(X^{(k)})$ [alternating group] is almost always maximal and determines the exceptions. The analogue of this for linear groups is: Show $SL(V)$ is "nearly" maximal in $A(L_k(V))$ where $L_k(V)$ is the collection of k-subspaces of V. In [4], Kantor and McDonough do this problem for $k = 1$. In this paper we treat a problem similar to these. Before we get to our results we first introduce some notation.

Suppose ϕ is a homomorphism from a group G to a group X, we will say that ϕ is *maximal* if $\phi(G)$ is a maximal subgroup of X. ϕ is said to be *nearly maximal* if whenever H is a proper subgroup of X and H contains $\phi(G)$, then H normalizes $\phi(G)$. Finally, for a prime p, we say ϕ is *p-maximal*, if for any proper subgroup H of X which contains $\phi(G)$, then a p-Sylow of $\phi(G)$ is a p-Sylow of H.

Now let V be a vector space of dimension $n \geq 3$ over a field $F = \mathsf{F}_{p^e}$ and for $k \leq n-1$, let $V_K = \Lambda^k(V)$. Set $G = SL(V)$, $G_k = SL(V_k)$ and define ϕ_k, a homomorphism from G into G_k by

$$(\phi_k g)(v_1 \wedge \ldots \wedge v_k) = (gv_1) \wedge (gv_2) \wedge \ldots \wedge (gv_k).$$

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