L^p MULTIPLIERS ON THE HEISENBERG GROUP

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1. INTRODUCTION

The Heisenberg group is the simplest example in the class of stratified groups. On these groups one can define a one-parameter family of anisotropic dilations and an homogeneous norm. Hence it is possible to extend to them many of the standard constructions of Euclidean spaces: singular integrals, homogeneous differential operators, Lipschitz classes etc. ([4], [10], [9]). However, except for a few instances where the representations of the group play a peripheral role, the noncommutative Fourier transform has not been so far a tool in this kind of harmonic analysis. More recently an attempt to make the Fourier transform on the Heisenberg group a usable tool in the study of the Schwartz space and of homogeneous differential operators has been made by Geller [6].

In this framework, using the theory of singular integrals on homogeneous spaces developed by Coifman, De Guzman and Weiss [2], we extend to the three-dimensional Heisenberg group the classical multiplier theorem of Hörmander.

We recall that Hörmander's theorem is stated in the following way [8].

THEOREM 1. *Let M be a function of a class* C^k in $\mathbf{R}^n \setminus \{0\}$, $k \geq \frac{n}{2} + 1$. *Assume that*

i) $M \in L^\infty(\mathbf{R}^n)$

ii) $$\sup_{R \in [0,+\infty)} R^{2|\alpha|-n} \int_{R<|\xi|\leq 2R} |\partial^\alpha M(\xi)|^2 d\xi \leq C$$

for all differential monomials ∂^α of order $|\alpha| \leq k$. Then the linear operator T_M *defined by*

$$T_M f(x) = \int e^{-2i\pi\langle x,\xi\rangle} M(\xi)\, \hat{f}(\xi)\, d\xi$$

is bounded on $L^p(R^n)$, $1 < p < \infty$.

This theorem has already been extended to $SU(2)$ by Coifman and Weiss [1] and to the group of Euclidean motions by Rubin [11].

In the next section we review some basic tools of the harmonic analysis on

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