NOTES ON ESSENTIALLY POWERS FILTRATIONS

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Dedicated to Louis Yager

1. INTRODUCTION

All rings in this paper are assumed to be commutative with an identity, and the undefined terminology is, in general, the same as that in [7].

In this paper a study is made of a special type of filtration on a ring A, an *essentially powers filtration* (*e.p.f.*—see 2.3 for the definition). Such filtrations have some interesting properties, and they are a useful and important concept since, as briefly noted after 2.3, each filtration on A can be closely approximated by them so knowledge about *e.p.f.*'s can be used to derive knowledge about more general filtrations on A.

In Section 2 three characterizations of an *e.p.f.* f on a Noetherian ring A in terms of the Rees ring $\mathscr{R}(A,f)$ are proved, and then some lattice theoretic properties of such filtrations are given in Section 3. In Section 4 it is shown that with each filtration g on an analytically unramified semilocal ring R there exist infinitely many filtrations on R that are associated with g and all of these are *e.p.f.*'s if and only if one of them is an *e.p.f.* Section 5 contains a number of characterizations of an analytically unramified semilocal ring in terms of *e.p.f.*'s, and in Section 6 it is shown that some of the results obtained in this paper are applicable to the Chain Conjecture in altitude three.

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2. ESSENTIALLY POWERS FILTRATIONS AND REES RINGS

In this section, after giving the basic definitions and known results that are needed in the remainder of this paper, three characterizations of an *e.p.f.* f on a Noetherian ring A are given in terms of the Rees ring $\mathscr{R}(A,f)$. We begin by recalling the definition of a filtration.

Definition 2.1. A *filtration* $f = \{A_n\}$ on a ring A is a descending sequence of ideals A_n of A such that $A_o = A$ and $A_n A_m \subseteq A_{n+m}$, for all n and m.

The next definition gives a partial order on the set of all filtrations on a ring.

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