CLASSIFICATION OF SO(3)-ACTIONS ON FIVE-MANIFOLDS WITH SINGULAR ORBITS

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INTRODUCTION

We describe the smooth $SO(3)$-actions on simply-connected, connected, closed five-dimensional manifolds admitting at least one orbit whose dimension is strictly less than the dimension of the principal orbits. We will show that such an $SO(3)$-manifold must be diffeomorphic to S^5, $S^2 \times S^3$, or the connected sum $kX_{-1} \# \ell M_2$, $k, \ell \geq 0$, where the five-manifold X_{-1} is diffeomorphic to the Wu-manifold $SU(3)/SO(3)$ and M_2 to the Brieskorn variety of the type (2,3,3,3,3).

Let $\mathscr{D}^i_{SO(3)}$ be the set of all smooth orientable $SO(3)$-manifolds of dimension five which admits no exceptional orbits (defined in I) and whose orbit spaces are diffeomorphic to the i-dimensional ball D^i, $i = 2$ or 3. Then using the techniques of Bredon [3], Hsiang and Hsiang [5] and Jänich [7], we can classify $\mathscr{D}^i_{SO(3)}$. Every manifold in $\mathscr{D}^2_{SO(3)}$ has two or three distinct orbit types; if exactly two distinct orbit types appear then the orbit structure is determined by the invariants $\{H, K; b\}$ where $SO(3)/H$ and $SO(3)/K$ are the orbit types and

$$b \in \Gamma = [S^1, N(H)/N(H) \cap N(K)]/\pi_0(N(H)/H);$$

Γ is isomorphic to the trivial group, Z_2 or Z_+ depending on the subgroups H and K. The pair (H,K) is $(\{e\}, SO(2))$, $(Z_k, SO(2))$, or $(D_k, N(SO(2))$. If $M \in \mathscr{D}^2_{SO(3)}$ admits three orbit types, one of them is the fixed point type and this M is determined by an equivalence class of a finite sequence of symbols $\{0,1,2\}$; the length of the sequence equals the number of the fixed points. For example, S^5 admits an $SO(3)$-action with two fixed points which is the one-point compactification of the irreducible linear action on R^5 [11]. We show that every manifold in $\mathscr{D}^2_{SO(3)}$ admitting two fixed points is equivalent to this action on S^5. A manifold in $\mathscr{D}^2_{SO(3)}$ with three fixed points is equivalent to the Wu-manifold $SU(3)/SO(3)$ with $SO(3)$ acting by the left coset-multiplication. For a manifold with four fixed points we have two equivalence classes. One of them is the equivariant connected sum

$$SU(3)/SO(3) \# SU(3)/SO(3),$$

and the other is M_2. From our classification theorem we know that there is exactly one $SO(3)$-action on M_2 and that this action has four fixed points, but this action is not natural in a sense that a manifold with four fixed points was constructed first and then it was identified as M_2 by Barden's classification (1). One might try to give a direct construction of this action. Every manifold in $\mathscr{D}^2_{SO(3)}$ with fixed points is simply-connected.

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