MAZUR MANIFOLDS

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1. INTRODUCTION

In [10] Mazur constructed a contractible 4-manifold whose boundary is a homology 3-sphere not equal to S^3. In this paper we investigate some generalized Mazur manifolds $W^{\pm}(\ell,k)$ gotten by adding a 2-handle to $S^1 \times B^3$ in certain ways. Consider the knots $K^{\pm}$ in $S^1 \times B^2 \subset S^1 \times S^2 = \partial(S^1 \times B^3)$ drawn below in Figure 1.

There are ℓ full twists (right-handed as drawn if $\ell > 0$, left-handed if $\ell < 0$) in $K^{\pm}$. The 0-framing on the normal bundle to $K^{\pm}$ is the one derived from the normal vector field which is tangent to a Seifert surface between $K^{\pm}$ and the curve $\gamma_{\pm}(= S^1 \times q$ where $S^1 \times q \cap K^{\pm} = \emptyset)$. Since $\pi_1(SO(2))$ acts on the normal bundle to $K^{\pm}$ in the obvious way, twisting k times, k determines a trivialization of the normal disk bundle which we use to attach a 2-handle to $S^1 \times B^3$, getting $W^{\pm}(\ell,k)$. Mazur's example ([10]) was $W^-(0,3) \approx W^+(0,0)$, (see section 2 for the diffeomorphism).

We consider the question: is $\gamma_{\pm}$ homotopically slice? That is, does $\gamma_{\pm}$ bound a *smoothly* imbedded disk in some contractible 4-manifold X^4 with $\partial X^4 = \partial W^{\pm}(\ell,k)$?

THEOREM 1. *γ_- is homotopically slice if and only if*

$$(\ell,k) = (0,0), \quad (4,1) \quad \textit{or} \quad (2,k).$$

THEOREM 1′. *γ_+ is homotopically slice if and only if*

$$(\ell,k) = (2,1), \quad (-2,0) \quad \textit{or} \quad (0,k).$$

Theorem 1′ follows from Theorem 1 because there is a diffeomorphism between $\partial W^-(\ell,k)$ and $\partial W^+(-\ell+2, -k+1)$ which takes γ_- to γ_+ (see Proposition 1, section 2).

Zeeman [13, page 357] suggested that no essential knot in the boundary of a contractible manifold is slice. Somewhat the opposite has turned out to be true (see [9] for some examples), for R. Fenn [3] showed that any circle in the boundary of a contractible manifold with a 2-dimensional spine is homotopic to one which is slice. However, some special cases are still interesting. It has been known for some time that γ_+ is slice in $W^+(0,k)$, for all k (the slice is drawn in section 5). However the same method cannot work for γ_- in $W^-(0,0)$ (see [9]). Considerable effort has not produced a slice. But as is known (section 5), γ_- is slice in some other contractible manifold W. W is h-cobordant to $W^-(0,0)$ (any contractible

Received November 24, 1976. Revision received March 21, 1978.

Michigan Math. J. 26 (1979).