PSEUDOCONVEXITY AND VALUE DISTRIBUTION FOR SCHUBERT ZEROES

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The distribution of zeroes of holomorphic sections in a Hermitian vector bundle was first studied using characteristic forms by Bott and Chern [2], and later by Cowen [5], Griffiths-King [8] and Stoll [15] [17]. In the general setting, let $f: X \to Y$ be a holomorphic map (where X, Y are complex spaces); assume in Y a reasonable set of subvarieties, $\mathfrak{A} = \{S_b\}_{b \in N}$, is given. One wishes to describe the typical behavior of the fiber $S_{b,f} = f^{-1}(S_b)$, $b \in N$. Assume X carries a pseudoconvex (respectively, pseudoconcave) exhaustion function; *i.e.*, a proper, C^∞ map $\varphi: X \to \mathbb{R}$ whose Levi form $L(\varphi) = dd^c\varphi \geq 0$ (respectively, $L(\varphi) \leq 0$) off a compact set. If $\{S_{b,f}\}$ is zero dimensional, suitable growth conditions or geometric properties of f imply that $S_{b,f} \neq \emptyset$ for almost all $S_b \in \mathfrak{A}$ (*e.g.* [3] [5] [6] [7] [14] [20]). If $\{S_{b,f}\}$ has positive dimension, in order to prove the same an additional closed, nonnegative form measuring the volume of $S_{b,f}$ was usually required ([9] [14] [17] [19]). In place of the latter hypothesis, one may assume there is a closed form $\theta \in A_2^{1,1}(X)$ such that outside a compact set, $\theta \geq 0$, $\theta \geq L(\varphi)$ and $\theta^m \not\equiv 0$ ($m = \dim X$). In terms of this θ the Casorati-Weierstrass type theorems can be established even in the case $L(\varphi)$ has eigenvalues of different signs. It is unknown, however, if such a θ exists for a given φ. If φ is strongly logarithmic pseudoconvex (in the sense of Griffiths-King-Stoll [8] [15]), the natural choice of θ is of course $L(\varphi)$. In this case, (under certain conditions) one can prove the equidistribution property: the valence of a generic S_b grows to infinity over suitable sequences of open sets at the same rate as the characteristic of f ([19,4.9]). Taking into account also the 0-convex exhaustion function of Andreotti-Grauert [1], a unified notion of pseudoconvexity which admits equidistribution seems to be of interest. To this end, the g-pseudoconvex, (g,y)-pseudoconvex as well as the g-pseudoconcave exhaustion functions are introduced in Section 1.

The equidistribution theorems are first proved for an admissible family $\mathfrak{A}$ in Y (Section 2). These can be applied to the case of Schubert zeroes of sections in a semi-ample vector bundle over Y (Section 3). The results obtained generalize those of Chern [3, p. 537] [4, 4.8], Cowen [5,7.1], Stoll [15,13.3,13.4] [17,4.6] and Wu [20, pp. 86–88].

1. EXHAUSTION FUNCTION AND G-PSEUDOCONVEXITY

For the basic notations the reader is referred to [19]. All complex spaces are assumed reduced, pure dimensional and countable at infinity. Let X be a complex space of dimension $m > 0$. Let $\varphi: X \to \bar{\mathbb{R}}\,[-\infty,\infty)$ be an exhaustion function; *i.e.*,

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