SIMPLY CONNECTED SURGERY OVER A RING

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1. INTRODUCTION

In [1] it is shown that the surgery obstructions for a simply connected problem are given by the signature, Kervaire invariant or invariants β_p lying in a certain 2-torsion group determined in [2]. The first two listed have been treated extensively in the literature. It is the purpose of this paper to compute the β_p-invariants of a normal map $f: M \to X$ in terms of M,X and the degree of f (section 3). We also deduce a product formula. Applications are given to Poincare complexes, homology spheres, singular manifolds and involutions.

2. SURGERY OBSTRUCTION GROUPS

Let R be a principal ideal domain. A map $f: X \to Y$ between path-connected, simply connected spaces is an R-*homotopy equivalence* if

$$f_{\#} \otimes 1: \pi_i(X) \otimes R \cong \pi_i(Y) \otimes R \qquad \text{for all } i.$$

Suppose $\pi_1 X = 0$ and $(X, \partial X)$ satisfies Poincare duality with coefficients in R, given by cap product with $[X,\partial X] \in H_n(X,\partial X)$. Let $f: (M,\partial M) \to (X,\partial X)$ be a map so that

(i) $(M,\partial M)$ is a compact n-manifold,

(ii) $f_*[M,\partial M]$ is a unit in $H_n(X,\partial X;R) \cong R$,

(iii) there is a bundle ξ over X and a bundle map $b: \nu_M \to \xi$ covering f,

and

(iv) $f | \partial M$ is an R-homotopy equivalence.

In [1] we construct a cobordism group $L_n(1;R)$ so that if $n \geq 5$, f is normally cobordant to an R-homotopy equivalence if and only if an obstruction in $L_n(1;R)$ vanishes. Let K be the set of primes p so that $R \otimes \mathbb{Z}/p = 0$; then $L_n(1;R) \cong L_n(\mathbb{Z}_K)$ where $\mathbb{Z}_K = \mathbb{Z}[1/p\colon p \in K]$, and $L_n(\mathbb{Z}_K)$ is K-theoretic group of [10]. The following is proved in [1]:

THEOREM 2.1. (i) $L_{2n+1}(\mathbb{Z}_K) = 0$

(ii) $L_{4n+2}(\mathbb{Z}_K) \cong \mathbb{Z}/2 \otimes \mathbb{Z}_K$

(iii) $L_{4n}(\mathbb{Z}_K) \cong W(\mathbb{Z}_K)$.

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