AN L_p ANALYTIC FOURIER-FEYNMAN TRANSFORM

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0. INTRODUCTION

In [1] Brue introduced an L_1 analytic Fourier-Feynman transform. In [3] Cameron and Storvick introduced an L_2 analytic Fourier-Feynman transform. In this paper we study an L_p analytic Fourier-Feynman transform for $1 \leq p \leq 2$. The resulting theorems extend the theory substantially (even in the cases $p = 1$ and $p = 2$) and indicate relationships between the L_1 and L_2 theories.

Before giving the basic definitions we fix some notation. $\mathbb{R}^n$ will denote n-dimensional Euclidean space, $\mathbb{C}$ the complex numbers and $\mathbb{C}^+$ the complex numbers with positive real part. $C_0(\mathbb{R}^n)$ will denote the $\mathbb{C}$-valued continuous functions on $\mathbb{R}^n$ which vanish at ∞. Wiener space, $C[a,b]$, will denote the $\mathbb{R}$-valued continuous functions on $[a,b]$ that vanish at a. Integration over $C[a,b]$ will always be with respect to Wiener measure. If Y and Z are Banach spaces, $L(Y,Z)$ will denote the space of continuous linear operators from Y to Z.

In this paper, as in [3], the term *Wiener measurable* will always mean measurable with respect to the uncompleted Wiener measure; that is measurable with respect to the σ-algebra of Borel sets in $C[a,b]$.

Definition. Let F be a functional such that the Wiener integral

$$J(\lambda) = \int_{C[a,b]} F(\lambda^{-1/2} x)\, dx \tag{0.1}$$

exists for almost all real $\lambda > 0$. If there exists a function $J^*(\lambda)$ analytic in the half-plane $\mathbb{C}^+$ such that $J(\lambda) = J^*(\lambda)$ for almost all real $\lambda > 0$, then we define this *essential analytic extension* of J to be the *analytic Wiener integral of F over* $C[a,b]$ *with parameter* λ and we write

$$\int_{C[a,b]}^{\text{anw}_\lambda} F(x)\, dx = J^*(\lambda) \qquad \text{for } \lambda \in \mathbb{C}^+. \tag{0.2}$$

Notation. For $\lambda \in \mathbb{C}^+$ and $y \in C[a,b]$ let

$$(T_\lambda F)(y) \equiv \int_{C[a,b]}^{\text{anw}_\lambda} F(x + y)\, dx. \tag{0.3}$$

Terminology. We shall say that two functionals F and G are equal *s-almost*

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