THE BEZOUT PROBLEM FOR A SPECIAL CLASS OF FUNCTIONS

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INTRODUCTION

For a holomorphic mapping $f: \mathbb{C}^2 \to \mathbb{C}P^2$ Cornalba and Shiffman [1] have shown that it is, in general, impossible to estimate the growth of $f^{-1}(W)$ in terms of the growth of f. Here $W \in \mathbb{C}P^2$ and $f^{-1}(W)$ is assumed discrete. The growth of $f^{-1}(W)$ is measured by counting the number of points in $f^{-1}(W) \cap \{|z| \leq r\}$. In this paper we give a class of functions E for which it is possible to measure the growth of $f^{-1}(W)$ in terms of the growth of f and another function M(r). It is hoped that M(r) will be easier to estimate than the error term S(r) of Griffiths [2].

1. NOTATION

Let $\omega = dd^c \log\|Z\|^2$ be the standard Kähler metric on $\mathbb{C}P^2$. Let $\tau = \log|z|^2$ be the exhaustion function on $\mathbb{C}^2$. If $\xi \in \mathbb{C}P^1$ let $\mathbb{C}_\xi$ be the corresponding line through the origin in $\mathbb{C}^2$. If $f: \mathbb{C}^2 \to \mathbb{C}P^2$ let $f_\xi = f|\mathbb{C}_\xi$. Let $W \in \mathbb{C}P^2$. Then $W = A \cap B$ (= intersection of perpendicular lines in $\mathbb{C}P^2$). Let

$$\omega_o = dd^c \log(|\langle Z,A\rangle|^2 + |\langle Z,B\rangle|^2)$$

in $\mathbb{C}P^2$. Let $\Lambda_W = \log[|Z|^2/(|\langle Z,A\rangle|^2 + |\langle Z,B\rangle|^2)](\omega + \omega_o)$. If $f: \mathbb{C}^2 \to \mathbb{C}P^2$ is holomorphic and $f^{-1}(W)$ discrete then we have the following functions from Nevanlinna theory:

$$n(W,r) = \operatorname{card}(\{|z| \leq r\} \cap f^{-1}(W))$$

$$N(W,r) = \int_0^r n(W,t)\, d\log t$$

(Here we assume $f(0) \neq W$, otherwise one must modify the counting function N(W,r).)

$$T_1(r) = \int_0^r \left\{ \int_{|z|\leq t} f^*\omega \wedge dd^c\tau \right\} d\log t$$

$$T_2(r) = \int_0^r \left\{ \int_{|z|\leq t} f^*\omega \wedge f^*\omega \right\} d\log t$$

$$S(W,r) = \int_{|z|\leq r} f^*\Lambda_W \wedge dd^c\tau.$$

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