ON REPRESENTATIONS OF ARTIN'S BRAID GROUP

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In [5], it is shown that the projective symplectic group $P\,Sp((n-2)/2, \mathbb{Z}_3)$ is an epimorphic image of B_n, Artin's Braid group on n strings. The method arises from machinery established by Hurwitz [10] for determining the action of B_n on branched coverings of the two-sphere. Redefining this action in terms of Fuchsian groups, a more direct proof of this result is obtained and the general method is shown to be allied to the methods of [8] of obtaining finite representations of the mapping class groups of related Fuchsian groups. These latter finite representations are discussed in Section 3. The link is provided in Section 2 by a general method of obtaining (infinite) symplectic representations of B_n, which is, in essence, a reformulation of results in [4].

1. PRELIMINARIES

A *Fuchsian group* is a discrete subgroup of $\mathscr{L} = PSL(2, \mathbb{R})$, the group of all conformal self-homeomorphisms of the upper half-plane U. A finitely-generated Fuchsian group of the first kind has a presentation of the form:

$$\text{(1)} \quad \begin{aligned} &\text{Generators: } e_1, e_2, \ldots, e_r, p_1, \ldots, p_s, a_1, b_1, \ldots, a_g, b_g \\ &\text{Relations: } e_i^{m_i} = 1\ (i = 1, 2, \ldots, r); \quad \prod_{i=1}^{r} e_i \prod_{j=1}^{s} p_j \prod_{k=1}^{g} [a_j, b_j] = 1 \end{aligned}$$

A Fuchsian group with presentation (1) has *signature* $(g; m_1, \ldots, m_r; s)$. The e_i are elliptic elements, the p_i parabolic and the a_i, b_i hyperbolic. The quotient space U/Γ takes the structure of a Riemann surface obtained from a compact surface of genus g by deleting s points. The covering $U \to U/\Gamma$ is branched over r points corresponding to the fixed points of $e_1, e_2, \ldots, e_r$ and the *periods* m_i give the order of branching at these points.

Γ has a fundamental region in U whose hyperbolic area $\mu(\Gamma)$ is given by

$$\text{(2)} \qquad \mu(\Gamma) = 2\pi\,[2(g-1) + \sum_{i=1}^{r}\left(1 - \frac{1}{m_i}\right) + s].$$

If Γ_1 is a subgroup of Γ of finite index n, then $\mu(\Gamma_1) = n\mu(\Gamma)$, which combined with (2) gives the Riemann-Hurwitz relation.

With Γ as at (1), an automorphism of Γ is called *type-preserving* if it maps parabolic elements into parabolic elements. Let F be a free group on $2g + r + s$

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