EXTREMAL PROPERTIES OF A CLASS OF SLIT CONFORMAL MAPPINGS

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1. INTRODUCTION

Let U denote $\{z : |z| < 1\}$ and H (U) the space of functions analytic in U endowed with the topology of uniform convergence on compact subsets of U. It is well known that H (U) is a locally convex topological space.

We will be concerned with the set $S \subset H(U)$ consisting of functions f,

$$f(z) = z + a_2 z^2 + \ldots,$$

that are univalent on U, and with several subsets of S. We denote by A the collection of functions $f \in S$ that map U onto the complement of a single analytic slit γ which has an asymptotic direction at ∞ and which possesses the $\pi/4$ property; i.e., the angle between the radius vector and the tangent vector at any point on γ is in absolute value smaller than $\pi/4$. By σ we denote the collection of support points of S; i.e., functions $f \in S$ that satisfy

$$\operatorname{Re} L(f) = \max_{g \in S} \operatorname{Re} L(g)$$

for some continuous linear functional L on H (U) which is nonconstant on S. Finally by E (S) and $E(\overline{co}S)$ we denote the set of extreme points of S and the set of extreme points of the closure of the convex hull of S respectively.

There are various relations between these classes of functions. For example, $\sigma \subset A$ is a result due to Pfluger [8] and later Brickman and Wilken [2]. Further, $E(\overline{co}S) \subset E(S)$ by a general argument for compact subsets of locally convex spaces [4; p. 440]. Also, Brickman [1] proved the striking result that if $f \in E(S)$, then f maps U onto the complement of a single Jordan arc along which $|w|$ increases to ∞.

In [6] Hengartner and Schober proved that if $f \in A$, $f(z) = z + a_2 z^2 + \ldots$, then $|a_2| > 1$. In the present note, we show that in fact $|a_2| > \sqrt{2}$ for functions in A and this result is best possible. In particular, $|a_2| > \sqrt{2}$ holds for $f \in \sigma$. However, we are unable to show that this result is best possible for σ and in fact we have been unable to find an $f \in \sigma$ with $|a_2| < 1.77$.

By a general result for locally convex spaces (see [3; p. 231]), $\bar{\sigma} \supset E(\overline{co}S)$. It therefore follows from our result that if $f \in E(\overline{co}S)$, $|a_2| \geq \sqrt{2}$. Using this

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