UNIQUENESS FOR SETS OF FUNCTIONS GENERATED BY MATRICES

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1. INTRODUCTION

In this paper we define admissible matrices. Using these matrices we define sets of admissible functions. That each of these sets of admissible functions is a complete orthonormal system for $L^1[[0, 1), \mathbb{C}]$ and that any admissible Fourier series of these functions (hereafter called Fourier series) converges almost everywhere are almost immediate from the definitions. The major result of this paper supplies sufficient conditions for uniqueness of admissible series. Some results on Walsh-like series found in [1], [3], [4], [5], and [8], among others, are similiar to our results.

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2. BASIC DEFINITIONS AND PROPERTIES

Definition 1. Let $A = (a_{ij})$ be an $n \times n$ matrix with complex entries and let $\bar{A} = (\bar{a}_{ij})$, where $\bar{a}_{ij}$ is the complex conjugate of a_{ij}. The nonsingular matrix A will be called an *admissible matrix* if $A\bar{A}^T$ is a diagonal matrix and $a_{1j} = 1$ for $1 \le j \le n$.

Note that a matrix composed of the characters of the cyclic group of order n arranged properly satisfies this condition.

Definition 2. Let $A = (a_{ij})$ be an $n \times n$ admissible matrix normalized so that $A\bar{A}^T = nI$. For $1 \le i \le n$ and $x \in [0, 1)$, let $g_i(x) = a_{ik}$, where $(k-1)/n \le x < k/n$ and $1 \le k \le n$. The functions $(g_i)_{i=1}^n$ defined in this way will be called the *admissible functions* derived from the matrix A. (In the future we will just refer to these functions as admissible functions.)

The family of complex-valued functions $(g_i)_{i=1}^n$ defined on $[0, 1)$ can be extended periodically to any interval (a, b). Note that $A\bar{A}^T = nI$ implies that the g_i are orthonormal.

Definition 3. Let $A = (a_{ij})$ be an $n \times n$ complex matrix, and let $B = (b_{ij})$ be an $m \times m$ complex matrix. $A * B$ will be used to denote the product of A by B, which is defined as follows. Let B_i be the ith row of B, and let $\otimes$ be the Kronecker product of two matrices; then

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