EXTRINSIC SPHERES IN KÄHLER MANIFOLDS, II

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1. INTRODUCTION

An n-dimensional submanifold M^n of an arbitrary Riemannian manifold is called an *extrinsic sphere* if it is umbilical and has parallel mean curvature vector $H \neq 0$. In the first part of this series, we have proved that a complete, simply connected, even-dimensional extrinsic sphere in any Kähler manifold is isometric to an ordinary sphere if its normal connection is flat. Moreover, we have proved that there exist no complete orientable extrinsic spheres of codimension 2 in any positively (or negatively) curved Kähler manifold.

On the other hand, it is known that there exist complete extrinsic spheres of codimension $p > m$ in 2m-dimensional complex projective space $P^{2m}(\mathbb{C})$ and complex sphere Q^{2m}. Since $P^{2m}(\mathbb{C})$ and Q^{2m} are Hermitian symmetric spaces of rank 1 and 2, and Hermitian symmetric spaces are the most important class of Kähler manifolds, it seems to be interesting to determine the codimensions of extrinsic spheres in all irreducible Hermitian symmetric spaces. In this paper, we shall study such codimensions.

We shall use the same notations as in the first part of this series [8], unless mentioned otherwise.

2. HERMITIAN SYMMETRIC SPACES

Let G/K be an irreducible Hermitian symmetric space with an involution τ, and let $\mathfrak{g}$ and $\mathfrak{k}$ be the Lie algebras of G and K, respectively. Then the eigenvalues of τ as a linear transformation of $\mathfrak{g}$ are 1 and -1, and $\mathfrak{k}$ is the eigenspace for 1. Let $\mathfrak{m}$ be the eigenspace for -1. Then $\mathfrak{g} = \mathfrak{k} + \mathfrak{m}$, and $\mathfrak{m}$ can be regarded as the tangent space of G/K at the origin. This decomposition of $\mathfrak{g}$ is called the *canonical decomposition* of $\mathfrak{g}$. On the Lie algebra $\mathfrak{g}$, the *Killing-Cartan form* ϕ is given by

$$\phi(X, Y) = \operatorname{tr}(\operatorname{ad} X \cdot \operatorname{ad} Y), \qquad \text{where } X, Y \in \mathfrak{g}.$$

The restriction of ϕ to $\mathfrak{m}$ defines a G-invariant Hermitian metric on G/K. It is well known that every G-invariant Hermitian metric is Kählerian and it is a constant multiple of the Killing-Cartan form.

The irreducible Hermitian symmetric spaces have been classified (up to constant multiples for the metric) by É. Cartan. Throughout this paper we shall assume that the maximal (respectively, minimal) holomorphic sectional curvatures are 1 (respectively, -1) for the Hermitian symmetric spaces of compact (respectively, noncompact) type. The dimensions, Ricci tensors $\tilde{S}$, metric tensors $\tilde{g}$, and their holomorphic sectional curvatures $\tilde{H}$ are given in Tables I and II [2, 7, 10]. We shall follow the notations of Helgason [9] for Lie groups and Hermitian symmetric spaces.

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