BROWDER-LIVESAY INDEX INVARIANT AND EQUIVARIANT KNOTS

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Let T be a differentiable fixed point free involution on a $(4n + 3)$-dimensional homotopy sphere Σ^{4n+3}, denoted by (T, Σ^{4n+3}). We know that (T, Σ^{4n+3}) always admits an invariant $(4n + 1)$-dimensional sphere S^{4n+1} [6, p. 81]. Furthermore, we may require that $(\Sigma^{4n+3}, S^{4n+1})$ be a simple knot [2].

As in [4] or [7], we may apply equivariant surgery in $X = \overline{\Sigma - (S \times D^2)}$ to obtain two 2n-connected Seifert submanifolds V_1 and V_2 of dimension $(4n + 2)$ such that $TV_1 = V_2$ and $\partial V_1 = S^{4n+1} \times \{0\}$, $\partial V_2 = S^{4n+1} \times \{\pi\}$. The set $V_1 \cup V_2$ divides X into two parts W_1 and W_2 with $TW_1 = W_2$.

Gluing V_1 and V_2 in the boundary of W_1 by the map $T: V_1 \to V_2$, we obtain a $(4n + 3)$-manifold Y. Let $\Sigma_1 = Y \cup S^{4n+1} \times D^2$ by some PL-homeomorphism $h: \partial Y \to S^{4n+1} \times S^1$. We see that (Σ_1, S^{4n+1}) is a simple $(4n + 3)$-knot. Choosing a basis $\{b_1, \cdots, b_m\}$ for $H_{2n+1}(V_1)$, we have a Seifert matrix A, which is the matrix for the mapping $j_1: H_{2n+1}(V_1) \to H_{2n+1}(W_1)$ with respect to the bases $\{b_i\}$ and $\{c_i\}$ determined by the Alexander duality. Let A^T be the transpose of A. Then $(-1)^{2n+2} A^T$ is the matrix for the mapping $j_2: H_{2n+1}(V_2) \to H_{2n+1}(W_1)$ with respect to the bases $\{T_* b_i\}$ and $\{c_i\}$.

From [3], we know that $A + (-1)^{2n+1} A^T = A - A^T$ is unimodular. But by using the same argument in [4], we can show that $A + A^T$ is also unimodular. For the involution (T, Σ^{4n+3}), Browder and Livesay defined an index invariant $\sigma(T, \Sigma^{4n+3})$. (For its definition, see [1] or [5].) The purpose of this note is to prove the following result.

THEOREM. $\sigma(T, \Sigma^{4n+3}) = \operatorname{index}(A + A^T)$.

Proof. In Σ^{4n+3}, we construct an invariant submanifold M of codimension 1 as follows:

$$M = V_1 \cup S^{4n+1} \times re^0 \cup S^{4n+1} \times re^{i\pi} \cup V_2 .$$

It is easy to see that M is 2n-connected, and $\{b_1, \cdots, b_m, T_* b_1, \cdots, T_* b_m\}$ forms a basis for $H_{2n+1}(M)$ by the natural inclusion $V_i \to M$, $i = 1$ or 2. M divides Σ^{4n+3} into two parts E_1 and E_2, with $TE_1 = E_2$. Under the inclusion $W_i \to E_i$, $i = 1$ or 2, we have a basis $\{c_i\}$ for $H_{2n+1}(E_1)$ and $\{T_* c_i\}$ for $H_{2n+1}(E_2)$.

Browder and Livesay [1] defined a symmetric bilinear form B on $H_{2n+1}(M)$ by $B(x, y) = x \cdot T_* y$. Since [3, p. 542] the $m \times m$ matrix $(b_i \cdot b_j) = A - A^T$, and $b_i \cdot T_* b_j = 0$, we see that B is represented by the matrix

$$\begin{pmatrix} 0 & A - A^T \\ A^T - A & 0 \end{pmatrix}$$

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