SYMMETRIC POWERS AND LEFSCHETZ NUMBERS

Benjamin Halpern

0. INTRODUCTION

In this note we point out several ramifications of a result of Dold's [1] concerning the Lefschetz numbers of the symmetric powers of a map. In [2], many consequences were deduced from a connection between the Lefschetz numbers of iterates of a map and a certain characteristic rational function. In [1], Dold provides a similar connection between the Lefschetz numbers of symmetric powers of a map and this same characteristic function. Consequently, a portion of [2] can be carried over to symmetric powers. Theorem 3.1 is an answer to a question raised by Dold in [1]. The author is indebted to A. Dold for stimulating conversations and for shortening some of the proofs.

1. NOTATION AND CONVENTIONS

We denote the rationals by $\mathscr{R}$. Homology is denoted by H and coefficients are taken in $\mathscr{R}$. If f is a linear self-mapping of a finite dimensional vector space over $\mathscr{R}$, then $X(f) = X(f; t)$ denotes its characteristic polynomial. Throughout this note Y is a compact CW-space and $g: Y \to Y$ is a continuous map.

$$\Lambda(g) = \operatorname{trace}((Hg)_{\text{even}}) - \operatorname{trace}((Hg)_{\text{odd}})$$

is the Lefschetz number of g, and $X(g) = X((Hg)_{\text{even}})/X((Hg)_{\text{odd}})$ is its characteristic rational function. $X(g)$ is an element of $\mathscr{R}(t)^*$, the multiplicative group of the field $\mathscr{R}(t)$ of rational functions over $\mathscr{R}$ in one indeterminate t. The Euler characteristic of Y is denoted by eY.

The nth symmetric power of Y is $P^{S(n)}Y = Y^n/\sim$, where two elements a and b of Y^n are equivalent under $\sim$ provided some permutation of the coordinates takes a to b. The nth symmetric power of g is the map $P^{S(n)}(g): P^{S(n)}Y \to P^{S(n)}Y$, induced from $\bar{g}: Y^n \to Y^n$, where $\bar{g}(y_1, \cdots, y_n) = (g(y_1), \cdots, g(y_n))$.

We denote the nth iterate of g by $g^n = g \circ g \circ \cdots \circ g$, n times.

2. PRELIMINARY RESULTS

First we state Theorem 5.9 of Dold [1].

THEOREM 2.1 (Dold). $\left[t^{eY} X\left(g; \frac{1}{t}\right)\right]^{-1} = \sum_{n=0}^{\infty} \Lambda(P^{S(n)}(g))\, t^n.$

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