SOME NEW PROPERTIES OF SUPPORT POINTS FOR COMPACT FAMILIES OF UNIVALENT FUNCTIONS IN THE UNIT DISK

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1. INTRODUCTION

Let $H(U)$ be the linear space of all analytic functions in the unit disk $U = \{z: |z| < 1\}$, with the topology of locally uniform convergence. Let $H'(U)$ be the topological dual space of $H(U)$, and $H_u(U)$ the set of all univalent functions in $H(U)$.

In this article we shall be interested in sets of univalent functions that lie in the intersection of two hyperplanes in $H(U)$; that is, in families

$$\mathscr{F} = \mathscr{F}(U, \ell_1, \ell_2, P, Q) = \{f \in H_u(U): \ell_1(f) = P, \ \ell_2(f) = Q\}$$

for fixed $\ell_1, \ell_2 \in H'(U)$ and $P, Q \in \mathbb{C}$. For example, one easily verifies that the special families

$$S = \{f \in H_u(U): f(0) = 0, \ f'(0) = 1\},$$

$$T = \{f \in H_u(U): f(p) = p, \ f(q) = q\}, \qquad p, q \in U, \ p \neq q,$$

are of this form.

In an earlier article [4], we characterized the families $\mathscr{F}$ that are nontrivial and compact. In particular, $\mathscr{F}(U, \ell_1, \ell_2, P, Q)$ is nonempty and compact if and only if

(a) $\ell_1(Q) \neq \ell_2(P)$

and

(b) $\ell_2(1)\,\ell_1(g) \neq \ell_1(1)\,\ell_2(g)$ for all $g \in H_u(U)$.

The families S and T are well known to be compact (the reader may also verify (a) and (b)). More generally, if ℓ_1 is any functional in $H'(U)$ that does not annihilate constants ($\ell_1(1) \neq 0$), we define the families

$$(1.1) \qquad \mathscr{S} = \{f \in H_u(U): \ell_1(f) = P, \ f'(q) = 1\} \qquad P \in \mathbb{C}, \ q \in U,$$

$$(1.2) \qquad \mathscr{T} = \left\{ f \in H_u(U): \ell_1(f) = P, \ \frac{f(p) - f(q)}{p - q} = 1 \right\} \quad P \in \mathbb{C}, \ p, q \in U \ (p \neq q).$$

Then $\mathscr{S}$ and $\mathscr{T}$ satisfy (a) and (b) and, consequently, are nonempty and compact. Actually, $\mathscr{S}$ is a limiting case of $\mathscr{T}$, corresponding to $p = q$. If $\ell_1(f) = f(0)$ and

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