COMPACTNESS OF λ-NUCLEAR OPERATORS

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1. INTRODUCTION

It is evident from the work of Persson and Pietsch [6] that the class of nuclear operators depends on ℓ_1, the space of absolutely convergent series. Replacing ℓ_1 by an arbitrary sequence space λ, we obtain a new class of operators called λ-nuclear, and we can pose questions motivated by known results in the case $\lambda = \ell_1$. The present work addresses the problem of under what restrictions the λ-nuclear operators are compact. Assuming that λ is a Banach space, Section 3 gives necessary and sufficient conditions on λ for λ-nuclear operators to be compact. Section 4 discusses a condition on the range of the operators that yields the same result.

2. PRELIMINARIES

We use λ to denote a *sequence space;* that is, a vector space whose elements are sequences of complex numbers, and we use $\lambda^\times$ for the *Köthe dual* of λ. ($\lambda^\times = \{b: \sum_{i=1}^\infty |a_i b_i| < \infty$ for all $a \in \lambda\}$.) A linear operator T between Banach spaces X and Y is *λ-nuclear* (respectively, *nuclear*) if

$$(1) \qquad Tx = \sum_{n=1}^\infty a_n \langle x, f_n \rangle y_n \quad \text{for all } x \in X,$$

where $\{a_n\}_{n=1}^\infty \in \lambda$ (respectively, ℓ_1), $f_n \in X'$ and $\sup_n \|f_n\| < \infty$, $y_n \in Y$ and $\{\langle y_n, g \rangle\}_{n=1}^\infty \in \lambda^\times$ for all $g \in Y'$. The series in (1) is required to converge in the norm topology on Y and (1) is referred to as a *λ-nuclear representation* for T. We will make use of some basic properties of λ-nuclear operators that have been discussed in sections (1.1) and (1.2) of [3].

All sequence spaces will be assumed to include ϕ, the set of finitely nonzero sequences, and to be *solid,* which means that $a \in \lambda$ if $b \in \lambda$ and $|a_i| \leq |b_i|$ for all i. Recall that a sequence space is a BK-*space* if it is a Banach space and each of the coordinate maps $a \to a_i$ is continuous. A sequence space λ is an AK-*space* if it is a topological vector space and $x = \lim_n P_n x$ for each $x \in \lambda$, where $P_n x = (x_1, x_2, \cdots, x_n, 0, \cdots)$. We say that λ is *perfect* if $\lambda = \lambda^{\times\times}$. The abbreviation $\lambda\mu$ will be used for the set of products $\{a_i b_i\}_{i=1}^\infty$ formed by taking $a \in \lambda$ and $b \in \mu$. We say that λ is *μ-invariant* if $\lambda = \mu\lambda$. Finally, c_0 denotes the BK-AK-space of sequences convergent to zero; ℓ_∞ is the BK-space of bounded sequences. Both have sup norm.

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