ON A PROPERTY OF INDICATORS OF SMOOTH CONVEX BODIES

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1. INTRODUCTION

It is well known that among all convex bodies in $\mathbb{R}^n$ the sphere enjoys several exceptional geometric properties (see [6]). Therefore, it may seem surprising that from the viewpoint of harmonic analysis the sphere also possesses certain unsatisfactory properties that are not shared by all convex bodies. Thus, for instance, it was shown recently by Ch. Fefferman [12] that the sphere is not a good multiplier in the L^p-theory of multiple Fourier series. Another problem in harmonic analysis where the sphere exhibits an unexpected behavior is the question of existence of solutions to convolution equations (see [3], [4], and Problem (B) below). To formulate this problem properly, we need some preliminaries.

We shall use the standard notation of the theory of distributions [17], [18]. In particular, $\mathscr{E}'$ is the convolution algebra of all distributions with compact support in $\mathbb{R}^n$. If $\Phi \in \mathscr{E}'$, we denote by $\hat{\Phi}$ the Fourier transform of Φ; that is, $\hat{\Phi}(\zeta) = \Phi(e^{-i\langle x,\zeta\rangle})$, where

$$x \in \mathbb{R}^n, \qquad \zeta = \xi + i\eta \in \mathbb{C}^n, \qquad \langle x, \zeta \rangle = \sum_{j=1}^{n} x_j \zeta_j .$$

By S^{n-1} we shall denote the unit sphere in $\mathbb{R}^n$, that is, the boundary of the unit ball $\mathbb{B}^n$ in $\mathbb{R}^n$. If K is a subset of $\mathbb{R}^n$, we denote by ch K the convex hull of K. By an *extreme point* of K we shall mean an extreme point of ch K. If A and B are subsets of $\mathbb{R}^n$, the sets $A \pm B$ are defined as $\{z \in \mathbb{R}^n: z = x \pm y,\ x \in A,\ y \in B\}$, with the convention $A \pm \emptyset = \emptyset$. It is easy to see that for each pair of distributions $\Phi, \Psi \in \mathscr{E}'$, the singular support of the convolution $\Phi * \Psi$ satisfies the relation

$$\text{ch sing supp } (\Phi * \Psi) \subseteq \text{ch sing supp } \Phi + \text{ch sing supp } \Psi ;$$

however, the inclusion cannot, in general, be replaced by equality (see [2], [3], [4], [18]). It is therefore natural to say that a distribution $\Phi \in \mathscr{E}'$ *propagates singularities* provided for every $\Psi \in \mathscr{E}'$ it satisfies the condition

$$\text{(1)} \qquad \text{ch sing supp}(\Phi * \Psi) = \text{ch sing supp } \Phi + \text{ch sing supp } \Psi .$$

Every distribution with this property is also *invertible*, that is, it satisfies the weaker condition (see [18])

$$\text{(2)} \qquad \text{ch sing supp } \Psi \subseteq \text{ch sing supp}(\Phi * \Psi) - \text{ch sing supp } \Phi \qquad (\forall \Psi \in \mathscr{E}').$$

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